A dynamical inflaton coupled to strongly interacting matter

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Based on 2302.06618 (PRL)
with
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Inflationary Cosmology

- Precise measurements of cosmic microwave background revealed that our universe is homogeneous, isotropic and almost flat.
- Can arise in big bang theory only from highly fine tuned initial conditions: horizon, flatness and magnetic-monopole problem of big bang cosmology.
- Period of exponential expansion of the early universe provides solution.
- Generated by inflaton field with appropriate potential.

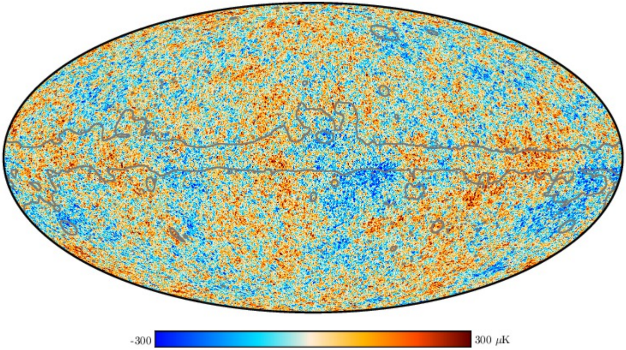
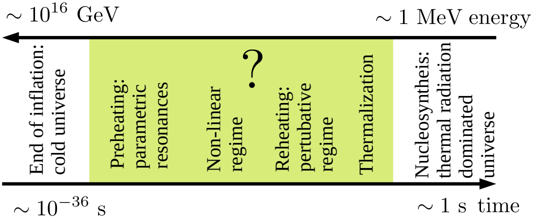


Image: Planck Collaboration arXiv:1807.06205 (Astronomy & Astrophysics)

Reheating after Inflation

- After the end of inflation the universe is typically[1] large, empty and cold.
- Need a mechanism that transfers inflaton energy to ordinary matter.
- Happens between the end of inflation and big bang nucleosynthesis, which is a time range that is poorly constrained by experiment and observation.
- Traditional picture: inflaton decay proceeds in different stages.



See, e.g., Kofman, Linde, Starobinsky arXiv:9704452 (PRD), review by Amin, Hertzberg, Kaiser, Karouby arXiv:1410.3808 (Int. J. Mod. Phys.), Lozanov arXiv:1907.04402 (lectures)

[1] An exception is warm inflation with particle production during inflation.
see, e.g., review by Bastero-Gil and Berera arXiv:0902.0521 (Int.J.Mod.Phys.)

Holographic Model for Inflationary Cosmology

Desired features:

- Early epoch of inflationary expansion and cooling.
- Mechanism that transfers inflaton energy to a holographic QFT.
- Thermalization of QFT matter in the late universe.

Caveats:

- Not unique: many possible choices for holographic theory, inflaton potential, coupling between different sectors, initial conditions, ...
- Holographic QFTs are strongly coupled: not QCD, hidden BSM sector?
- So far only a proof of principle of the method with a specific setup rather than a phenomenological model of our universe.

Gauge/Gravity Duality

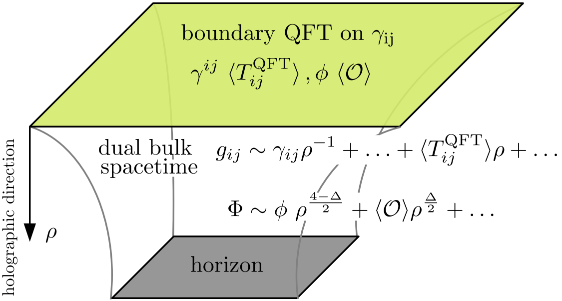
Mathematical relation between strongly coupled quantum field theory (QFT) and higher dimensional gravity in asymptotic anti-de Sitter (AdS) spaces

$$Z_{\mathrm{QFT}}[\gamma,\phi] = \int [\mathcal{D}g]_\gamma [\mathcal{D}\Phi]_\phi e^{-S_{\mathrm{hol}}[g,\Phi]} \;\leftrightarrow\; \mathcal{L}_{\mathrm{CFT}} + \Lambda^{4-\Delta}\mathcal{O}\,. \tag{1}$$

Holographic dictionary maps near-boundary expansion of bulk fields to sources and vacuum expectation values (VEV) of operators in the dual QFT

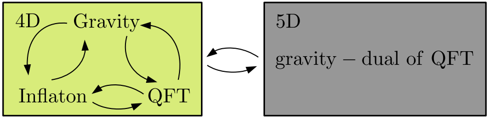
$$\langle T_{ij}^{\mathrm{QFT}}\rangle = \frac{2}{\sqrt{-\gamma}}\frac{\delta S_{\mathrm{hol}}}{\delta\gamma^{ij}}\,,\quad \langle\mathcal{O}\rangle = \frac{1}{\sqrt{-\gamma}}\frac{\delta S_{\mathrm{hol}}}{\delta\phi}\,. \tag{2}$$

Standard AdS/CFT assumes fixed boundary conditions γ_{ij}, ϕ (sources).



Framework

Goal: Build a framework that is able to describe dynamically the mutual interaction between gravity, the inflaton and a strongly coupled holographic QFT in a self-consistent way.



Minimal ingredients: Einstein-Hilbert+Inflaton action, a holographic QFT and some interaction that couples them non-minimally

$$S = S_{\mathrm{EH+inf}} + S_{\mathrm{hol}} + S_{\mathrm{int}}\ . \quad (3)$$

Many possibilities for S: we choose a specific setup that is guided by simplicity and intuition about inflationary cosmology.

Non-standard AdS/CFT: Coupling bulk and boundary dynamics requires AdS/CFT with mixed boundary conditions.

Compere, Marolf arXiv:0805.1902 (Class.Quant.Grav)

AdS/CFT with Dynamic Boundary Conditions

Promote boundary conditions to dynamical fields:

$$Z_{\rm ind} = \int \mathcal{D}\gamma\mathcal{D}\phi Z_{\rm QFT}[\gamma,\phi] = \int \mathcal{D}g\mathcal{D}\Phi e^{-S_{\rm hol}[g,\Phi]}\,, \tag{4}$$

$$\delta_g S_{\rm hol} = \int_{\mathcal{M}} {\rm d}x^5\sqrt{-g}\,{\rm EOM}^{(\rm g)}_{\rm bulk}\,\delta g^{\mu\nu} + \int_{\partial\mathcal{M}} {\rm d}x^4\sqrt{-\gamma}\frac{1}{2}\langle T^{\rm QFT}_{ij}\rangle\delta\gamma^{ij}\,, \tag{5}$$

$$\delta_\Phi S_{\rm hol} = \int_{\mathcal{M}} {\rm d}x^5\sqrt{-g}\,{\rm EOM}^{(\Phi)}_{\rm bulk}\,\delta\Phi + \int_{\partial\mathcal{M}} {\rm d}x^4\sqrt{-\gamma}\langle\mathcal{O}\rangle\delta\phi\,. \tag{6}$$

There are different ways to make the action stationary

- Dirichlet boundary conditions: $\delta\gamma_{ij} = 0$, $\delta\phi = 0$
- Neumann boundary conditions: $\langle T^{\rm QFT}_{ij}\rangle = 0$, $\langle\mathcal{O}\rangle = 0$
- Mixed boundary conditions:

$$\frac{1}{2}\langle T^{\rm QFT}_{ij}\rangle + \frac{\delta S_{{\rm bdry},\gamma}}{\delta\gamma^{ij}} = 0\,, \quad \langle\mathcal{O}\rangle + \frac{\delta S_{{\rm bdry},\phi}}{\delta\phi} = 0 \tag{7}$$

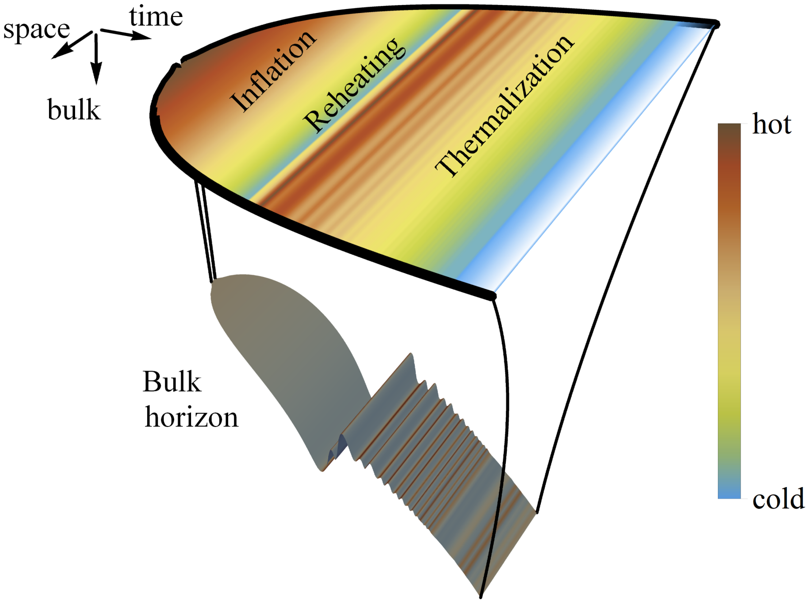
Compere, Marolf arXiv:0805.1902 (Class.Quant.Grav)

This talk: impose Einstein-Hilbert-inflaton action + interaction term

$$S_{{\rm bdry},\gamma,\phi} = S_{\rm EH+inf}[\gamma_{ij},\phi] + S_{\rm int}[\gamma_{ij},\phi]\,. \tag{8}$$

Generalization of previous works with dynamical boundary gravity (without inflaton) and for semi-holographic glasma (class. Yang-Mills) evolution (without gravity).

CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2011.08194 (JHEP)
CE, Mukhopadhyay, Preis, Rebhan, Soloviev arXiv:1806.01850 (JHEP)



Cover of PRL Volume 130, Number 25, 23 June 2023

Boundary Gravity and Inflaton

Einstein-Hilbert+Inflaton action

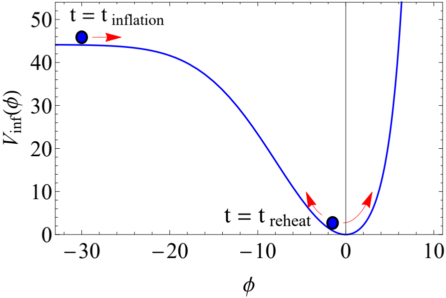
$$S_{\mathrm{EH+inf}} = \int \mathrm{d}^4x\sqrt{-\gamma}\left(\frac{R}{2\kappa_4} - \frac{1}{2}\gamma^{ij}\partial_i\phi\partial_j\phi - V_{\mathrm{inf}}(\phi)\right) . \qquad (9)$$

Friedmann–Lemaître–Robertson–Walker type metric

$$\mathrm{d}s^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2 . \qquad (10)$$

Coleman-Weinberg type inflaton potential with fixed v_0 and ϕ_m such that $V_{\mathrm{inf}}(0) = V'_{\mathrm{inf}}(0) = 0$

$$V_{\mathrm{inf}}(\phi) = v_0 + \tfrac{9}{8}\, e^{\frac{2}{3}(\phi-\phi_m)} - 45\, e^{-\frac{1}{50}(\phi-\phi_m)^2} . \qquad (11)$$



Holographic QFT

5D Einstein-dilaton gravity with non-trivial potential

$$S_{\text{bulk}} = \frac{2}{\kappa_5} \int \mathrm{d}^5 x \sqrt{-g} \left(\frac{1}{4}\mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V_{\text{bulk}}(\Phi) \right) . \tag{12}$$

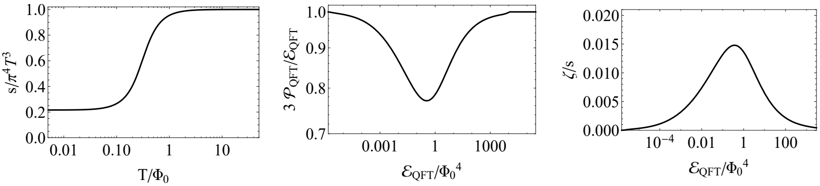
Potential inspired by GPPZ model: holographic SUGRA dual of $\mathcal{N} = 1$ SYM

Girardello, Petrini, Porrati, Zaffaroni, arXiv:hep-th/9909047 (Nucl.Phys. B)

$$V_{\text{bulk}}(\Phi) = \frac{1}{L^2} \left(-3 - \frac{3\Phi^2}{2} - \frac{\Phi^4}{3} + \frac{11\Phi^6}{96} - \frac{\Phi^8}{192} \right) . \tag{13}$$

Smooth RG-flow between IR and UV fixed points and broken conformal symmetry between.

Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Olivan, Sopuerta, Triana, Zilhao arXiv:1603.01254 (JHEP)



Fixing Ambiguities

The bare bulk action $S_{\rm bulk}$ needs to be remormalized by adding appropriate counter-terms that render the holographic action $S_{\rm hol}$ finite on-shell

$$S_{\rm hol} = S_{\rm bulk} + S_{\rm ct} \,. \tag{14}$$

Counter terms are unique up to some finite contributions

$$S_{\rm ct} = \frac{1}{\kappa_5}\int {\rm d}^4x\sqrt{-\gamma}\left[\left(-\frac{1}{8}R - \frac{3}{2} - \frac{1}{2}\Phi_{(0)}^2\right) + \frac{1}{2}\left(\log\rho\right)\mathcal{A} + \left(\alpha\mathcal{A} + \beta\Phi_{(0)}^4\right)\right]. \tag{15}$$

Anomaly due to curved boundary metric and broken conformal symmetry

$$\mathcal{A} = \frac{1}{16}(R^{ij}R_{ij} - \frac{1}{3}R^2) - \frac{1}{2}\left(\partial_i\Phi_{(0)}\partial^i\Phi_{(0)} + \frac{1}{6}R\Phi_{(0)}^2\right) \,. \tag{16}$$

Arbitrary constants α and β renormalize the bare gravitational coupling and the cosmological constant in the boundary theory

$$\frac{1}{\kappa_4} = \frac{1}{\kappa_{4,\rm bare}} + \frac{\alpha}{96\,\kappa_5} \,, \tag{17}$$

$$\frac{\Lambda_4}{\kappa_4} = \frac{\Lambda_{4,\rm bare}}{\kappa_4} - \frac{\beta}{1024\,\pi} \,. \tag{18}$$

We choose the "supersymmetric" renormalization scheme $\alpha = 0$, $\beta = \frac{1}{16}$.

CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)

Interaction Term

The interaction term implements a direct coupling between the inflaton and the scalar VEV ($\langle \mathcal{O} \rangle = \mathcal{O}_{\rm QFT}/\kappa_5$) of the holographic QFT

$$S_{\rm int} = \int {\rm d}^4 x \sqrt{-\gamma}\; U(\phi)\; \mathcal{O}_{\rm QFT}\,. \tag{19}$$

In the QFT the inflaton hence acts as a source for the scalar operator $\mathcal{O}$, where the source is given by the boundary value of the bulk scalar $\Phi_{(0)} = U(\phi)$.

Different choices for $U(\phi)$ are possible, but in the following we assume linear coupling for simplicity

$$U(\phi) = \lambda\phi\,, \quad \lambda = {\rm const.}\,. \tag{20}$$

Energy-Momentum Tensor

The total EMT in the boundary theory consists of three parts

$$T_{ij} = T^{\rm inf}_{ij} + \mathcal{T}^{\rm QFT}_{ij} + T^{\rm int}_{ij} = {\rm diag}\left(\mathcal{E}, \mathcal{P}, \mathcal{P}, \mathcal{P}\right) \, . \tag{21}$$

Inflaton contribution is given by the standard scalar field expression

$$T^{\rm inf}_{ij} = \partial_i \phi \partial_j \phi - \gamma_{ij} \left(\frac{1}{2} \partial_k \phi \partial^k \phi + V_{\rm inf} \right) \, . \tag{22}$$

Holographic part follows from variation of the renormalized holography action

$$\mathcal{T}^{\rm QFT}_{ij} = \frac{2}{\kappa_5 \sqrt{-\gamma}} \frac{\delta S_{\rm hol}}{\delta \gamma^{ij}} = {\rm diag}\left(\mathcal{E}_{\rm QFT}, \mathcal{P}_{\rm QFT}, \mathcal{P}_{\rm QFT}, \mathcal{P}_{\rm QFT}\right) \tag{23}$$

EMT and scalar VEV are related by anomaly corrected Ward identities

$$\gamma^{ij} \mathcal{T}^{\rm QFT}_{ij} = \mathcal{E}_{\rm QFT} - 3 \, \mathcal{P}_{\rm QFT} = -U(\phi) \, \mathcal{O}_{\rm QFT} + \mathcal{A} \, , \tag{24}$$

$$\nabla^i \mathcal{T}^{\rm QFT}_{ij} = -\partial_j \, U(\phi) \, \mathcal{O}_{\rm QFT} \, . \tag{25}$$

The direct coupling between the inflaton and the holographic sector gives

$$T^{\rm int}_{ij} = -\frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\rm int}}{\delta \gamma^{ij}} = U(\phi) \, \mathcal{O}_{\rm QFT} \, \gamma_{ij} \, . \tag{26}$$

The total EMT is covariantly conserved on-shell: $\nabla_i T^{ij} = 0$.

Initial Value Problem

4D Friedmann + scalar field equation for the inflaton that is coupled to $\mathcal{O}_{\rm QFT}$

$$H(t)^2 = \frac{\kappa_4}{3}\mathcal{E}(t)\,, \tag{27}$$

$$\frac{a''(t)}{a(t)} = -\frac{1}{2}\left(\kappa_4\mathcal{P}(t) + H(t)^2\right)\,, \tag{28}$$

$$\phi''(t) = \partial_\phi\, U(\phi(t))\,\mathcal{O}_{\rm QFT}(t) - 3H(t)\phi'(t) - \partial_\phi V_{\rm inf}(\phi(t))\,, \tag{29}$$

where $H(t) = a'(t)/a(t)$ is the Hubble rate.

5D Einstein–Klein–Gordon equations

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\,g_{\mu\nu} = 2\partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(2V_{\rm bulk} + (\partial\Phi)^2\right)\,, \tag{30}$$

$$\Box_g\Phi = \frac{\partial V_{\rm bulk}}{\partial\Phi}\,, \tag{31}$$

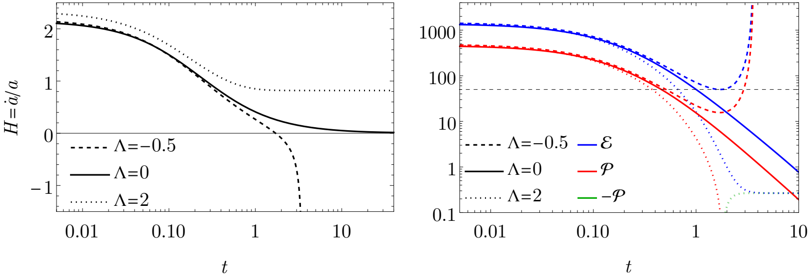
with Friedmann–Lemaître–Robertson–Walker metric imposed at the boundary

$$\mathrm{d}s^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2\,. \tag{32}$$

Coupled system has to be solved numerically with appropriate parameters $(\kappa_5, \kappa_4, \lambda)$ and initial conditions $(\mathcal{E}_{\rm QFT}^{\rm ini}, \phi_{\rm ini}, \phi'_{\rm ini}, \Phi_{\rm ini}(r))$ to get the time evolution of the scale factor $a(t)$, the inflaton $\phi(t)$ and the EMT $T_{ij}(t)$.

Static Inflaton

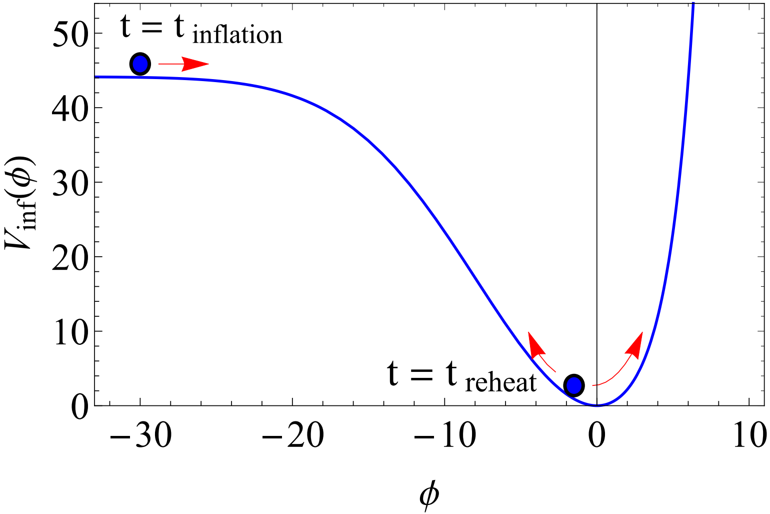
- Let's first see what happens when we freeze the inflaton: $\phi(t) = \text{const.}$.
- Pick some value for cosmological constant Λ and initialize QFT.
- Depending on the value of Λ, the universe ends up in a Big Crunch (Λ < 0), in flat space (Λ = 0) or in de Sitter (Λ > 0).
- de Sitter solution has some Casimir energy $\mathcal{E}_{\mathrm{dS}} = -\mathcal{P}_{\mathrm{dS}}$.



CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)

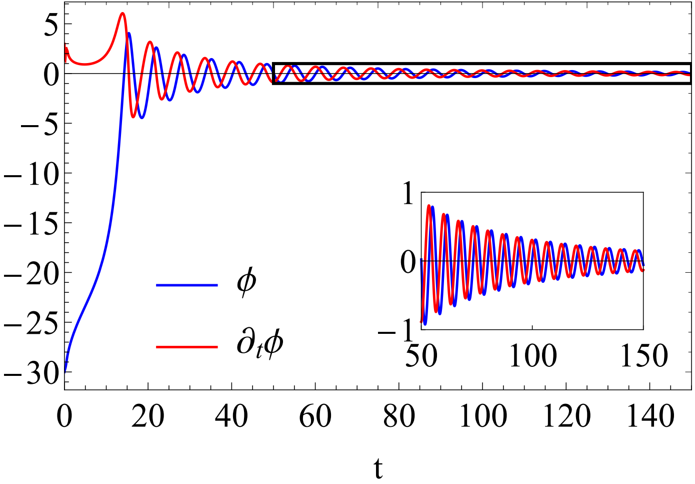
Dynamic Inflaton

- Initially slow-rolling inflaton induces exponentially expanding universe.
- Inflation ends when the inflaton rolls into the bottom of the potential.
- Oscillations around the minimum reheat the QFT.



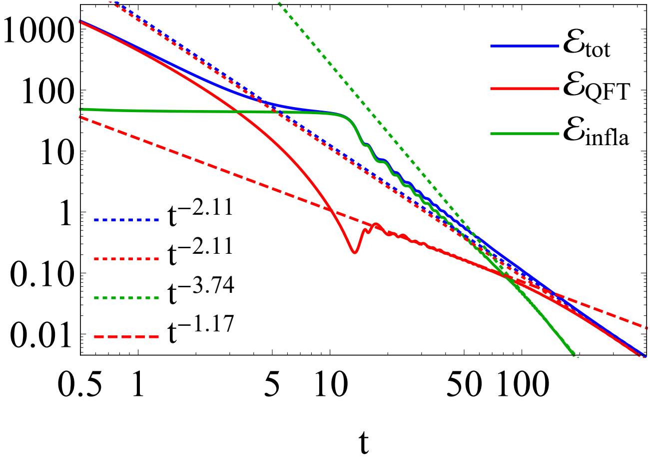
Inflaton

- Early phase ($t \lesssim 3$) dominated by QFT energy, after which the universe enters a period of constant exponential expansion.
- At arround $t \approx 14$ the inflaton reaches the bottom of the potential where it oscillates rapidly and sources the energy for the QFT.



Energy Density

- QFT energy is dominant until $t \approx 3$, then the inflaton dominates and until it reaches the bottom of the potential ($t \approx 14$).
- Inflaton oscillations reheat the QFT from $\mathcal{E}_{\mathrm{QFT}} = 0.21$ at $t = 13.5$ to a subsequent maximum of $\mathcal{E}_{\mathrm{QFT}} = 0.64$ at $t = 17.3$.
- Reheating continues: relatively slow scaling $\mathcal{E}_{\mathrm{QFT}} \propto t^{-1.17}$ of the QFT

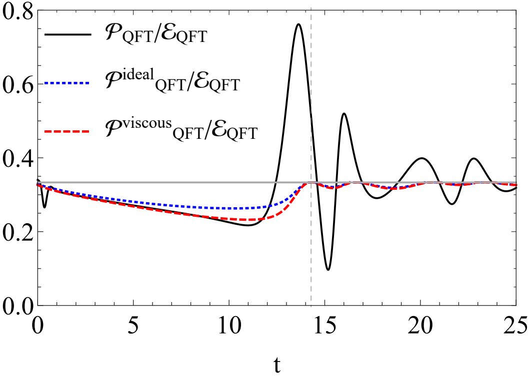


QFT Pressure

- After a short far-from-equilibrium stage, the system is well described by hydrodynamics until the inflaton sources the QFT out of equilibrium

$$\mathcal{P}^{\rm viscous}_{\rm QFT}(t) = \mathcal{P}^{\rm ideal}_{\rm QFT}(\mathcal{E}_{\rm QFT}(t)) - 3H\zeta(\mathcal{E}_{\rm QFT}(t)) + \mathcal{O}(H^2)\,. \qquad (33)$$

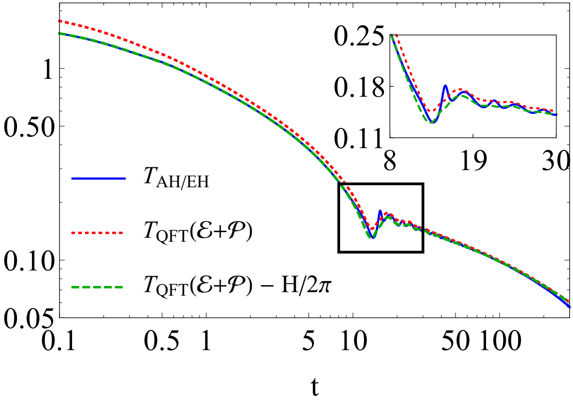
- QFT evolves from the UV into the non-conformal regime and then back to the UV fixed point where $\mathcal{P}_{\rm QFT} = \mathcal{E}_{\rm QFT}/3$.



as also seen in CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP), Casalderrey-Solana, CE, Mateos, van der Schee arXiv:2011.08194 (JHEP)

Temperature

- The QFT temperature can be computed from surface gravity of the bulk apparent horizon: $T_{\rm AH} = \kappa/2\pi$.
- Except for a short far-from-equilibrium period during reheating, the apparent and even horizon temperatures are numerically indistinguishable.
- Hydrodynamic approximation with EOS works well after subtracting de Sitter temperature of the cosmological horizon: $T_{\rm dS} = H/2\pi$.
  Consistent with exact CFT solution by Buchel, Heller, Noronha arXiv:1603.05344 (PRD)



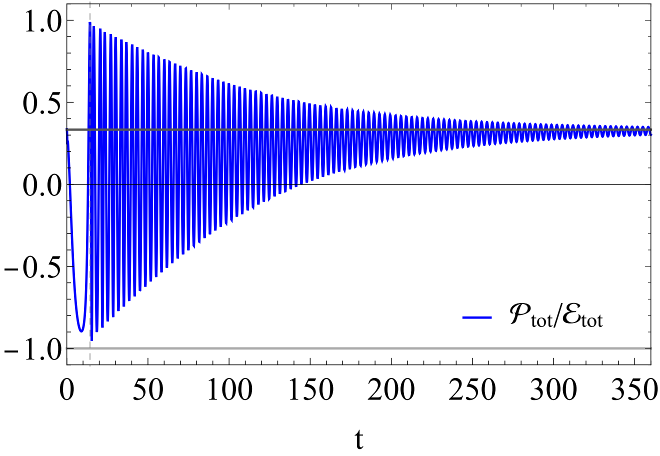
Summary

- The period between the end of inflation and big-bang nucleosynthesis is not well theoretically understood and observationaly constrained.
- Constructed a holographic framework that mimics the essential features of inflationary cosmology: inflationary phase, reheating and thermalization.
- Based on a combined action that assumes semi-classical boundary gravity and the standard holographic dictionary with mixed boundary conditions.
- So far we have solved only one specific simple setup: lots of room to explore different holographic QFTs, couplings and inflaton potentials.
- Mathematica code available with all formulas (equations of motion, asymptotic series, holographic renormalization, . . .) and numeric scheme.

https://sites.google.com/view/wilkevanderschee/public-codes

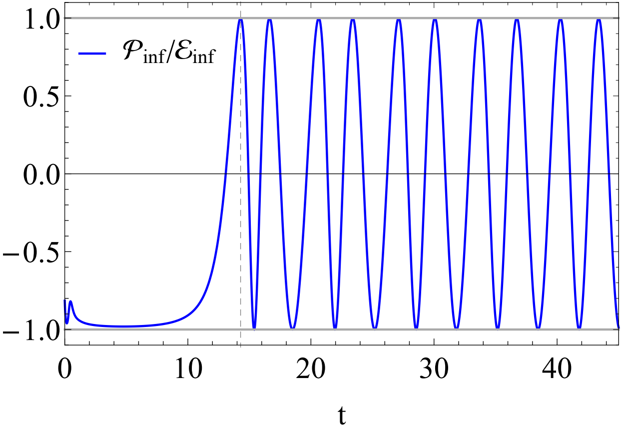
Total Pressure

- The total pressure is initially dominated by the QFT, then by the inflaton and at late times again by the reheated QFT, which is then close to the conformal UV fixed point $\mathcal{P}_{\rm tot} \approx \mathcal{E}_{\rm tot}/3$.



Inflaton Pressure

- Inflaton acts initially like a pure cosmological constant $\mathcal{P}_{\rm inf} = -\mathcal{E}_{\rm inf}$.



Explicit Expressions for Holographic VEVs

$$ds^2 = L^2 \frac{\mathrm{d}\rho^2}{4\rho^2} + \bar{g}_{ij}(\rho,x)\mathrm{d}x^i\mathrm{d}x^j \,, \quad (34)$$

$$\begin{aligned} \bar{g}_{ij}(\rho,x) &= \frac{1}{\rho}\Big[\gamma_{ij}(x) + \rho\,\gamma_{(2)ij}(x) + \rho^2\gamma_{(4)ij}(x) \\ &+ \rho^2 \log\rho\, h_{(4)ij}(x) + O(\rho^3)\Big]\,, \quad (35) \\ \Phi(\rho,x) &= \rho^{1/2}\Big[\Phi_{(0)}(x) + \rho\,\Phi_{(2)}(x) + \rho\,\log\rho\,\psi_{(2)}(x) + O(\rho^2)\Big]. \quad (36) \end{aligned}$$

$$\begin{aligned} \langle T_{ij}^{\mathrm{QFT}}\rangle &= \frac{2}{\kappa_5}\Big\{\gamma_{(4)ij} + \frac{1}{8}\left[\mathrm{Tr}\gamma_{(2)}^2 - (\mathrm{Tr}\gamma_{(2)})^2\right]\gamma_{ij} \\ &- \frac{1}{2}\gamma_{(2)}^2 + \frac{1}{4}\gamma_{(2)ij}\mathrm{Tr}\gamma_{(2)} + \frac{1}{2}\partial_i\Phi_{(0)}\partial_j\Phi_{(0)} \\ &+ \left(\Phi_{(0)}\Phi_{(2)} - \frac{1}{2}\Phi_{(0)}\psi_{(2)} - \frac{1}{4}\partial_k\Phi_{(0)}\partial^k\Phi_{(0)}\right)\gamma_{ij} \\ &+ \alpha\left(\mathcal{T}_{ij}^{\gamma} + \mathcal{T}_{ij}^{\phi}\right) + \left(\frac{1}{18} + \beta\right)\Phi_{(0)}^4\gamma_{ij}\Big\}. \quad (37) \end{aligned}$$

$$\langle\mathcal{O}\rangle = \frac{2}{\kappa_5}\left[(1-4\alpha)\psi_{(2)} - 2\Phi_{(2)} - 4\beta\Phi_{(0)}^3\right]. \quad (38)$$

Bulk Equations of Motion

$$\mathrm{d}s^2_{\mathrm{bulk}} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2 \mathrm{d}\vec{x}^2 \,, \qquad (39)$$

$$\Phi = \Phi(r,t)\,, \qquad (40)$$

$$S'' = -\frac{2}{3} S \left(\Phi'\right)^2 \,, \qquad (41)$$

$$\dot{S}' = -\frac{2\dot{S}S'}{S} - \frac{2SV}{3}\,, \qquad (42)$$

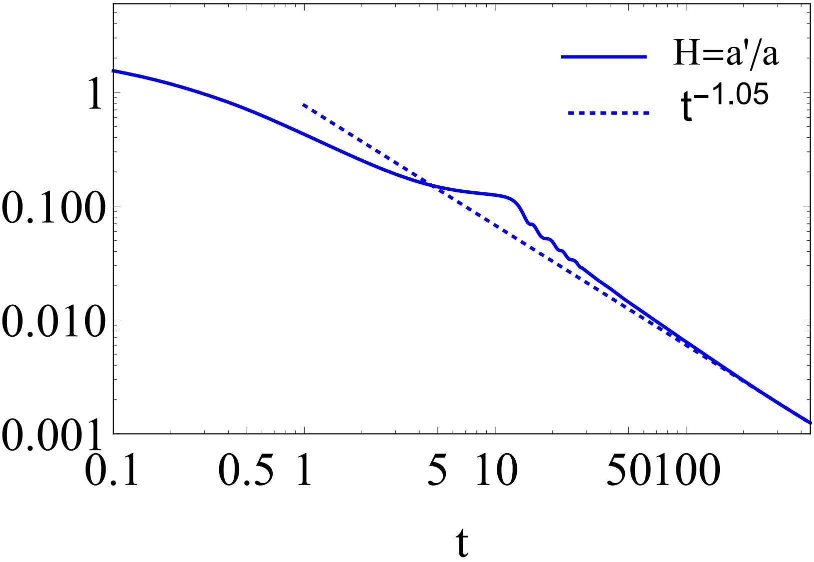
$$\dot{\Phi}' = \frac{V'}{2} - \frac{3\dot{S}\Phi'}{2S} - \frac{3S'\dot{\Phi}}{2S}\,, \qquad (43)$$

$$A'' = \frac{12\dot{S}S'}{S^2} + \frac{4V}{3} - 4\dot{\Phi}\Phi'\,, \qquad (44)$$

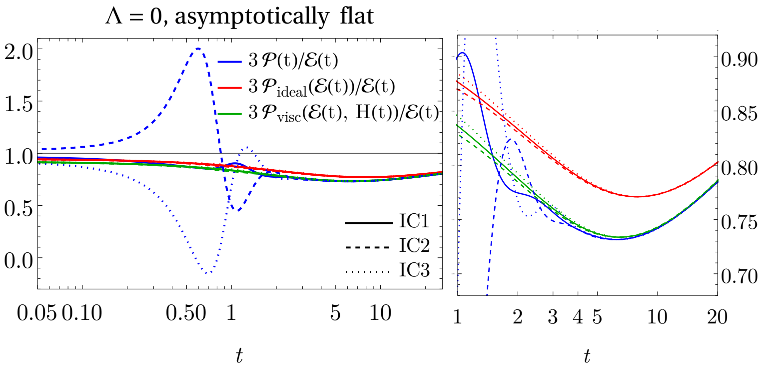
$$\ddot{S} = \frac{\dot{S}A'}{2} - \frac{2S\dot{\Phi}^2}{3}\,, \qquad (45)$$

$$f' \equiv \partial_r f\,, \quad \dot{f} \equiv \partial_t f + \frac{1}{2} A \partial_r f\,. \qquad (46)$$

Hubble Rate



Hydrodynamization with frozen inflaton



CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)