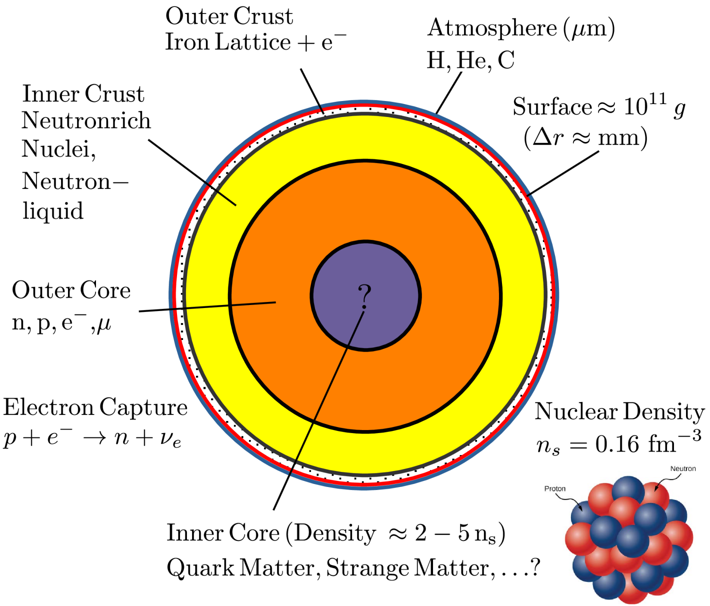
# On the Sound Speed in Neutron Stars

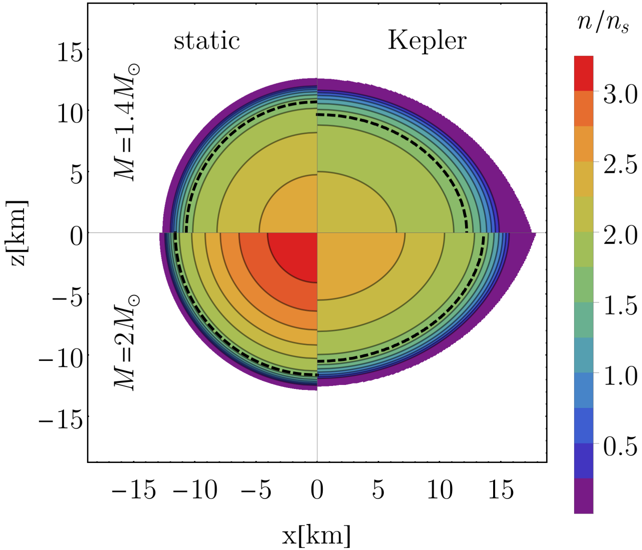
Christian Ecker

CRC-TR 211 Meeting
Frankfurt, 14 December 2022

Based on 2203.14974 (ApJL), 2207.04417 (ApJL), 2209.08101 and 2211.00018
with Sinan Altiparmak, Jin-Liang Jiang, and Luciano Rezzolla


Outer Crust
Iron Lattice + $e^-$
Atmosphere ($\mu$m)
H, He, C
Inner Crust
Neutronrich
Nuclei,
Neutron–
liquid
Surface $\approx 10^{11}\ g$
($\Delta r \approx$ mm)
?
Outer Core
n, p, $e^-$, $\mu$
Electron Capture
$p + e^- \to n + \nu_e$
Nuclear Density
$n_s = 0.16\ \mathrm{fm}^{-3}$
Neutron
Proton
Inner Core (Density $\approx 2 - 5\,\mathrm{n_s}$)
Quark Matter, Strange Matter, . . . ?

# Density Profile




Demircik, CE, Järvinen 2009.10731 (ApJL)

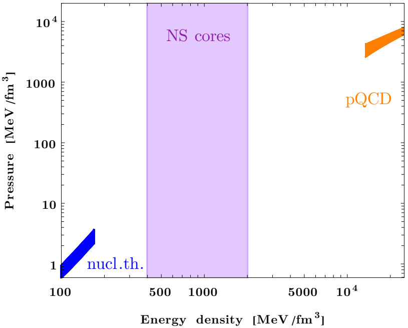
# Equation of State

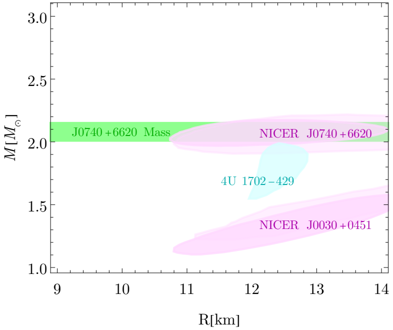
Relates properties of dense matter to properties of neutron stars.

$$p = p(e, \ldots) \xrightarrow{\text{Einstein Eq.}} M(R)$$

Constrained by theory and neutron star observations:

- Causality & thermodynamic stability: $0 < c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} < 1$
- Direct mass and radius measurements: $M_{\mathrm{TOV}} \gtrsim 2\,M_\odot$, $11\,\mathrm{km} \lesssim R \lesssim 14\,\mathrm{km}$
- First GW detection from a BNS merger GW170817: $\tilde{\Lambda} \lesssim 720$



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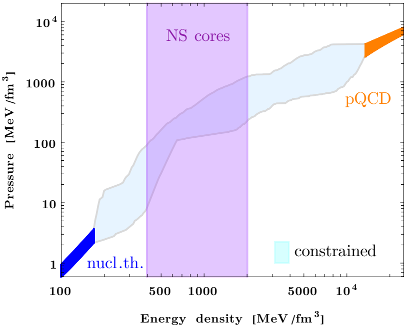
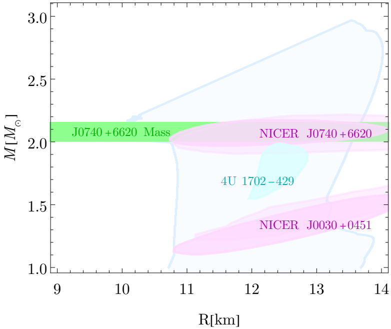


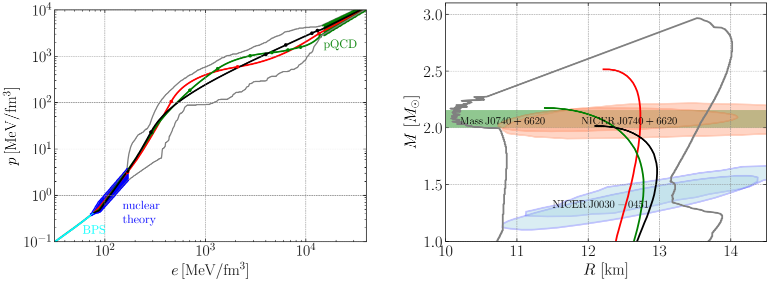
# Agnostic Approach

- Impose boundary conditions from nuclear theory and perturbative QCD.
  Hebeler, Lattimer, Pethick, Schwenk 1303.4662 (ApJ);
  Fraga, Kurkela, Vuorinen 1311.5154 (ApJL)
- Connect with random, but causal and thermodynamically consistent EOS.

$$c_s^2(\mu) = \frac{(\mu_{i+1}-\mu)c_{s,i}^2 + (\mu-\mu_i)c_{s,i+1}^2}{\mu_{i+1}-\mu_i}\,,\; n(\mu) = n_1\, e^{\int_{\mu_1}^{\mu} \frac{\mathrm{d}\mu'}{\mu' c_s^2(\mu')}}\,,\; p(\mu) = p_1 + \int_{\mu_1}^{\mu} \mathrm{d}\mu'\, n(\mu')\,.$$

Annala, Gorda, Kurkela, Nättilä, Vuorinen 1903.09121 (Nature Physics)

- Constrain with pulsar and binary neutron star observations.




Lots of recent work:
...; Annala, Gorda, Kurkela, Nättilä, Vuorinen 2105.05132 (PRX);
Komoltsev, Kurkela 2111.05350 (PRL); Altiparmak, CE, Rezzolla (ApJL) 2203.14974;
Gorda, Komoltsev, Kurkela 2204.11877; CE, Rezzolla 2207.04417 (ApJL); ...

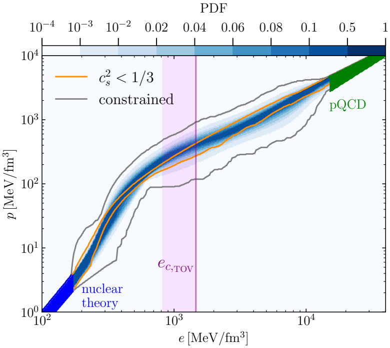
# Sampling the EOS

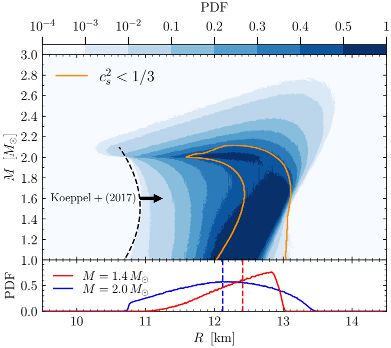
- Statistics of millions of EOS models that pass theory+astro constraints.
- Mass bound of generic (sub-conformal) models: $M_{\rm TOV} < 3\,(2.1)\,M_\odot$.
- Radius estimates (+ error bars) for typical stars:

$$R_{1.4} = 12.42^{+0.52}_{-0.99}\,\mathrm{km}\,,\ R_{2.0} = 12.11^{+1.11}_{-1.23}\,\mathrm{km}\,(95\%\,\mathrm{confidence})\,.$$

- Remarkable agreement with $R_{min}(M)$ from threshold mass calculations.

Koeppel, Rezzolla 1901.09977 (ApJL)




Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

# Sound Speed in Dense Matter

- Adiabatic speed of sound[1] provides a measure for the stiffness of matter

$$c_s^2 = \left(\frac{\partial p}{\partial e}\right)_s \,, \quad 0 \leqslant c_s^2 \leqslant 1 \,.$$

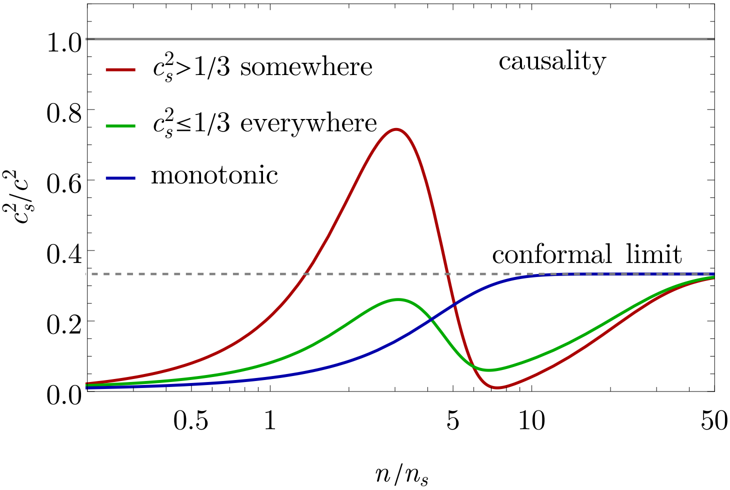
- Relevant for neutron star properties: stiff (soft) matter has large (small) sound speed and results in large (small) maximal mass and radius.
- "Slope" of the Equation of State (EOS) only known in certain limits:

$$c_s^2(n) = \begin{cases} \ll 1 & \forall\, n \ll n_s \,, \\ \quad ? & \forall\, n \gtrsim n_s \,, \quad n_s = 0.16\,\mathrm{fm}^{-3} \,, \\ \to 1/3 & \forall\, n \gg n_s \,. \end{cases}$$

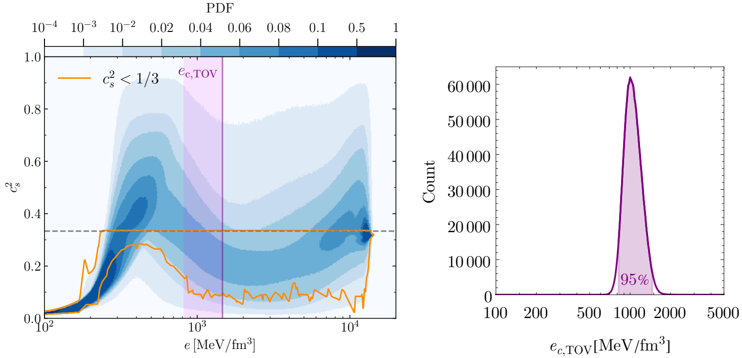
- Difficult open problem: What is $c_s^2(n)$ in QCD at neutron star densities?

[1]We use units where we set the speed of light $c = 1$.

# Several Different Possibilities



# Sound Speed in Dense Matter




- Local maximum $c_s^2 > 1/3$ below maximal central densities of TOV stars

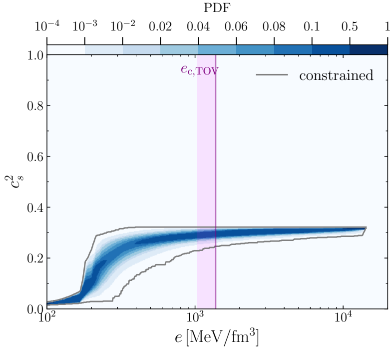
$$e_{c,\mathrm{TOV}} = 1064^{+399}_{-244}\,\mathrm{MeV/fm^3}\ (95\%\,\mathrm{confidence})\,.$$

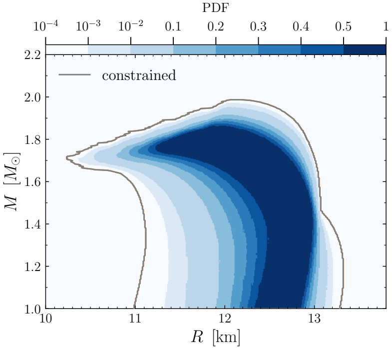
- $c_s^2 \leqslant 1/3$ at all densities possible, but only a small fraction (0.03%).
- Lower bound $c_s^2 \gtrsim 0.2$ of sub-conformal models around $e \approx 400\,\mathrm{MeV/fm^3}$.

Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

# Monotonic Sound Speed

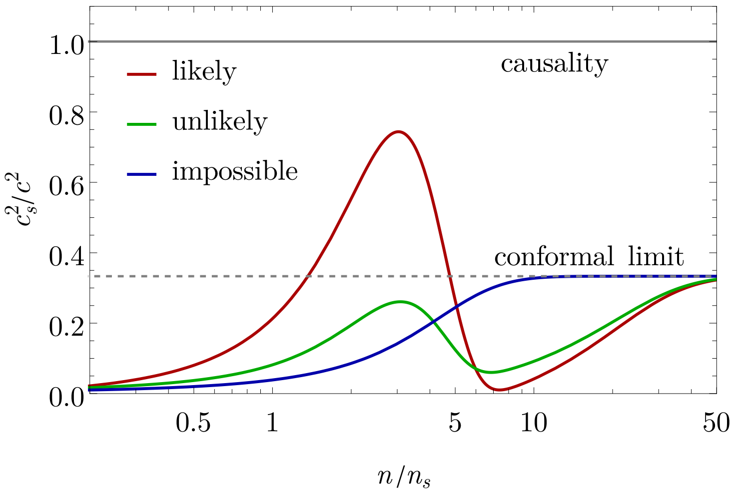
- Separate calculation with more than $10^7$ models.
- Monotonic by construction, but no lower mass bound.
- Can only reach $M_{TOV} < 2\,M_\odot$.



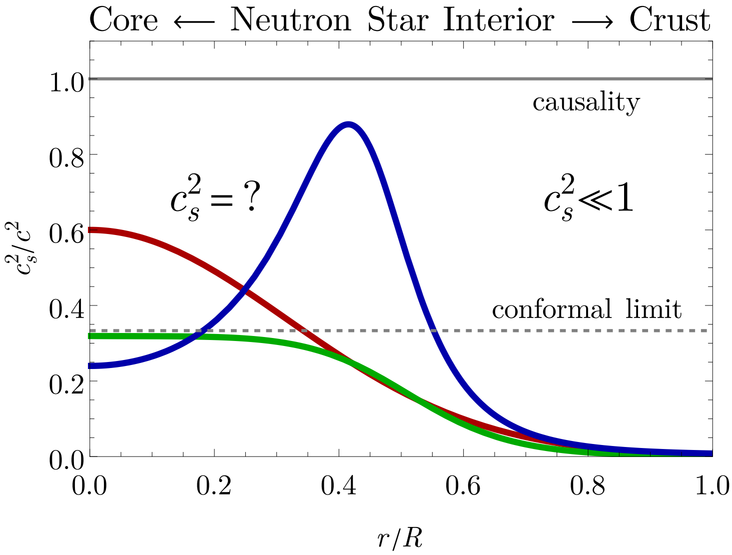




Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

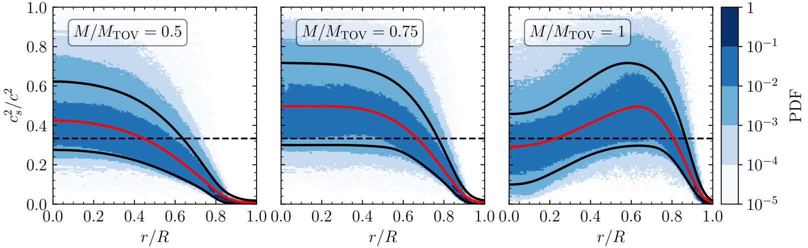
# Fewer Possibilities



# Speed of Sound inside Neutron Stars?



# Speed of Sound inside Neutron Stars




Median (red) and 95% intervals (black) can be accurately described by

$$c_s^2(x) = \left(\alpha e^{\beta x^2} + \gamma e^{\delta(x-\epsilon)^2}\right)\left[1 - \tanh\left(\zeta x - \eta\right)\right], \quad x \equiv r/R\,.$$

- **Light Stars:** stiff core, soft outer layer.
- **Typical Stars:** large stiff core.
- **Massive Stars:** soft core, stiff outer layer.

CE, Rezzolla 2207.04417 (ApJL)

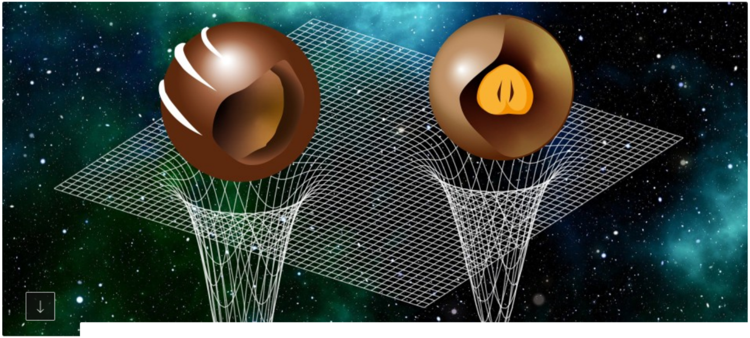
Frankfurter Allgemeine
ZEITUNG FAZ.NET



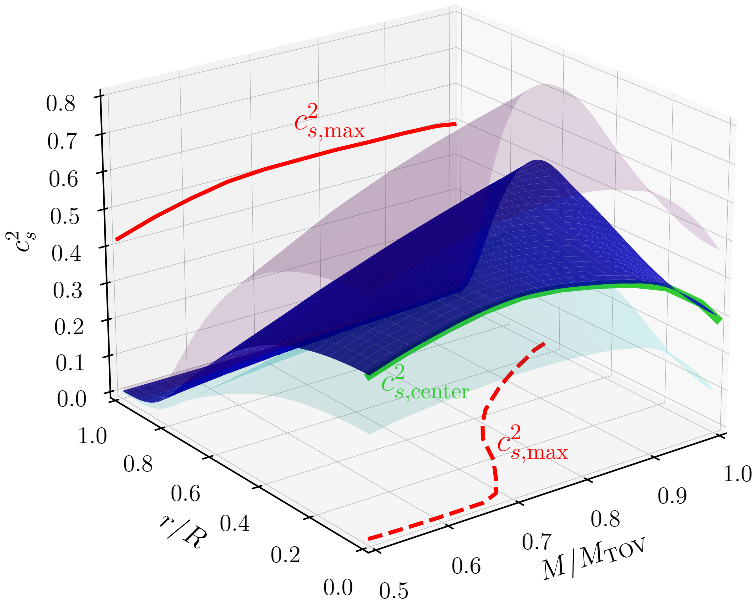
VON PHYSIKERN ERRECHNET

# Was Neutronensterne mit Pralinen gemeinsam haben

VON SASCHA ZOSKE - AKTUALISIERT AM 17.11.2022 - 06:47

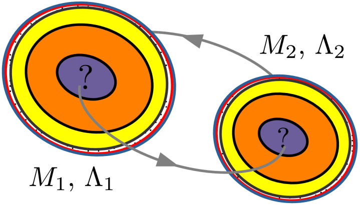


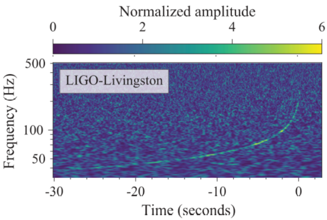
**Neutronensterne sind winzig klein, aber ungeheuer dicht. Frankfurter Forscher haben errechnet, wie sie aufgebaut sind – und ziehen einen süßen Vergleich.**




CE, Rezzolla 2207.04417 (ApJL)

# Chirp Mass and Binary Tidal Deformability




- Binary tidal deformability encodes EOS properties via $\Lambda_i$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12M_2 + M_1)\, M_1^4 \Lambda_1 + (12M_1 + M_2)\, M_2^4 \Lambda_2}{(M_1 + M_2)^5}\ , \qquad \Lambda_i = \tfrac{2}{3} k_2 \left(R_i/M_i\right)^5 \ .$$

- Chirp mass can be extracted accurately from inspiral GW strain frequency

$$\mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = \left( \frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right)^{3/5} \ .$$

- GW170817 only "good" event so far:

$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002} M_\odot, \quad q = M_2/M_1 > 0.7, \quad \tilde{\Lambda} \lesssim 720\,.$$

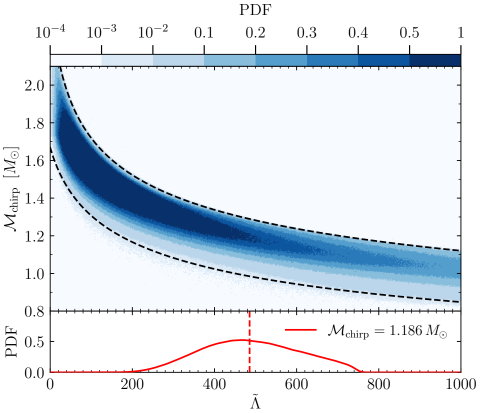
LIGO/Virgo 1710.05832 (PRL), 1710.05834 (ApJL), 1805.11579 (PRX)

# Binary Tidal Deformability

- More than $10^8$ binary configurations with $M_{1,2} \in [0.5M_\odot, M_{\rm TOV}]$.

$$\tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}^c_{chirp} \longrightarrow \tilde{\Lambda}_{1.186} = 485^{+225}_{-211}\,,$$

$$a = -50(-20),\ b = 500(1800),\ c = -4.5(-5.0)\,.$$




Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

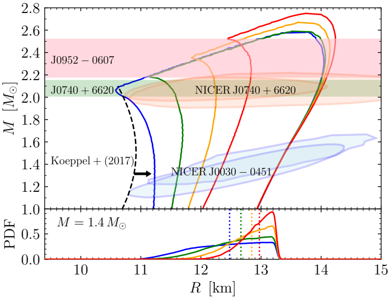
see also: Zhao, Lattimer 1808.02858 (PRD)

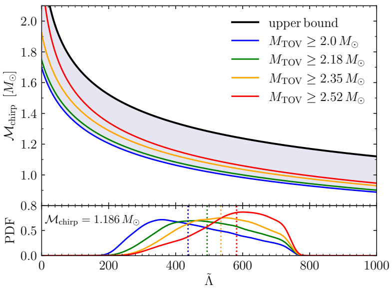
# Black-Widow Binary Pulsar PSR J0952-0607

- Most massive NS known: $M = 2.35 \pm 0.17\, M_{\odot}$, but large uncertainties!

  Romani, Kandel, Filippenko, Brink, Zheng 2207.05124 (ApJL)

  also: PSR J2215+5135 ($M = 2.27^{+0.17}_{-0.15}\, M_{\odot}$) Linares, Shahbaz, Casares 1805.08799 (ApJ)
- Large mass constraints dominate over radius constraints by NICER.




| $M_{\rm TOV}$ $[M_{\odot}]$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $\geqslant 2.00$ | $-50$ | 600 | 4.7 |
| $\geqslant 2.18$ | $-45$ | 650 | 4.6 |
| $\geqslant 2.35$ | $-40$ | 750 | 4.5 |
| $\geqslant 2.52$ | $-20$ | 800 | 4.4 |

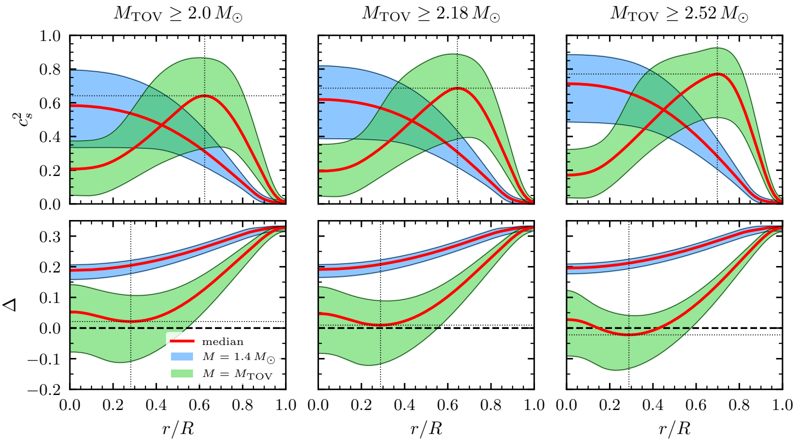
CE, Rezzolla 2209.08101

# Sound Speed and Conformal Anomaly

$$\Delta := \frac{1}{3}\frac{g_{\mu\nu}T^{\mu\nu}}{e} = \frac{1}{3} - \frac{p}{e}\,, \quad -\frac{2}{3} \leqslant \Delta \leqslant \frac{1}{3}\,.$$

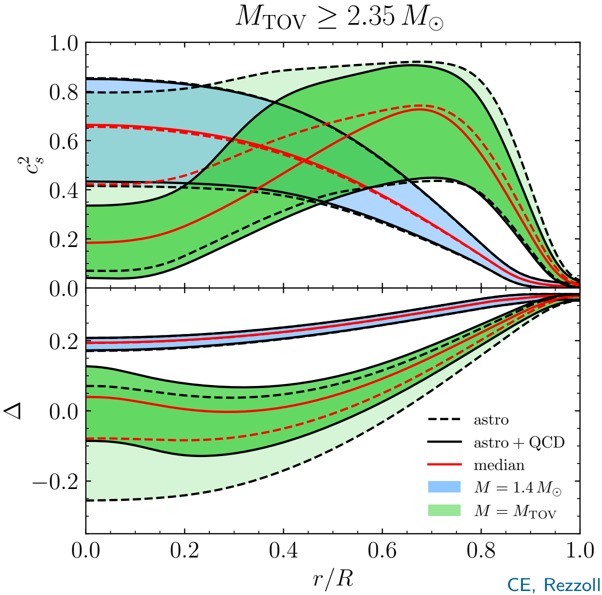
Fujimoto, Fukushima, McLerran, Praszalowicz 2207.06753

Marczenko, McLerran, Redlich, Sasaki 2207.13059




CE, Rezzolla 2209.08101

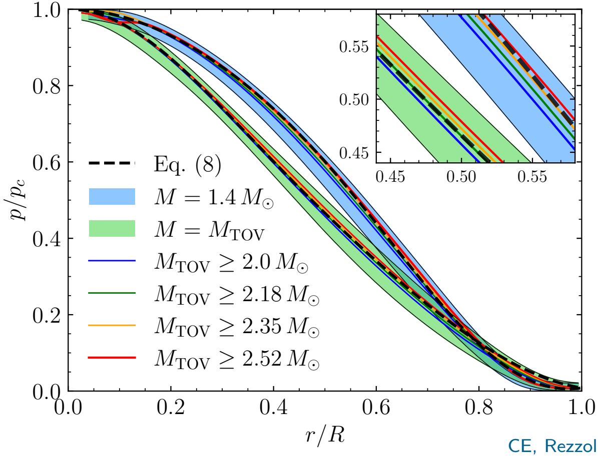
# Impact of QCD




CE, Rezzolla 2209.08101

# Quasi-Universality of the Pressure Profile

$$\frac{p(x)}{p_c} = e^{\alpha x^2} + e^{\alpha}\frac{\cos\beta x - 1}{\cos\beta - 1}\,, \qquad x \equiv r/R\,.$$



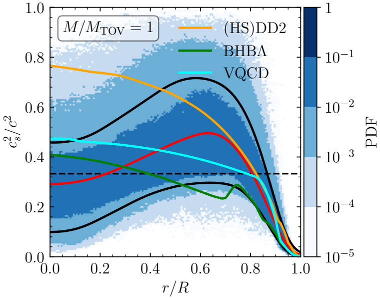

CE, Rezzolla 2209.08101

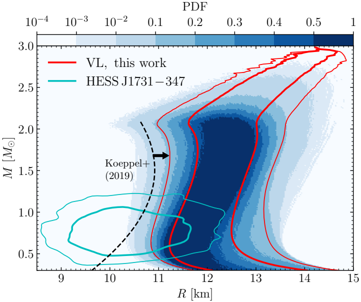
# Two Puzzles

- *Is there a micro-physical model that can explain the agnostic sound-speed profile inside neutron stars?*
- *What is the reason for the strong tension between the HESS J1731-347 measurement and model agnostic M-R distribution?*

$$M = 0.77^{+0.20}_{-0.17}\, M_\odot\,, \quad R = 10.4^{+0.86}_{-0.78}\,\mathrm{km}\,.$$

Doroshenko, Suleimanov, Pühlhofer, Santangelo (Nature Astronomy Oct.2022)




CE, Rezzolla 2207.04417 (ApJL), Jiang, CE, Rezzolla 2211.00018

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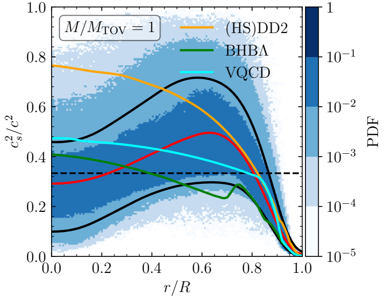
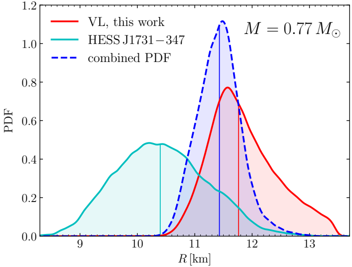
- *What is the reason for the strong tension between the HESS J1731-347 measurement and model agnostic M-R distribution?*
- Combined estimate with agnostic Bayesian analysis:

$$M = 0.77\,M_\odot\,, \quad R = 11.43^{+0.64}_{-0.60}\,\mathrm{km}\,.$$




CE, Rezzolla 2207.04417 (ApJL), Jiang, CE, Rezzolla 2211.00018

# Summary

**This talk:**

- Basic properties (+ error-bars) of the cold EOS and neutron stars can be estimated from constraint sampling: $p(e)\,, M(R)\,, c_s^2\,, \tilde{\Lambda}(\mathcal{M}_{\rm chirp})\,, \ldots$
- Using $r/R$ allows to sample the NS interior: $c_s^2(r/R)$, $\Delta(r/R)$, ...
- Light stars have a stiff core and soft outer layers, massive stars have soft cores and stiff outer layers.
- Can provide additional constraints for micro-physical EOS model building and help to identify quasi-universal relations.

**Ongoing and future work:**

- Model specific sampling: RMFT, V-QCD, ...
- Sound speed and universal relations in rotating stars.
- Sampling binary neutron star observables: post-merger frequencies, ...

# Building blocks of agnostic EOSs

- Lowest densities ($0 - 0.5\, n_s$): Baym-Pethick-Sutherland (BPS) EOS

  Baym, Pethick, Sutherland (1971)

- Low densities ($0.5 - 1.2\, n_s$): $p(n) = K\, n^\Gamma\,, \quad \Gamma \in [1.77, 3.23]$

  Hebeler, Lattimer, Pethick, Schwenk arXiv:1303.4662

- High densities ($1.2 - 40\, n_s$): speed of sound parametrization

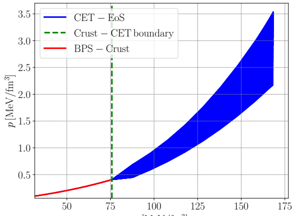
  Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:1903.09121

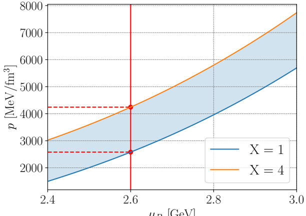
- Highest densities ($\gtrsim 40\, n_s$): perturbative QCD results

$$p(\mu) = \frac{3}{4\pi^2}\left(\frac{\mu}{3}\right)^4 \left(c_1 - \frac{d_1 X^{-\nu_1}}{\mu/GeV - d_2 X^{-\nu_2}}\right)\,, \quad X \in [1,4]\,, \qquad (1)$$

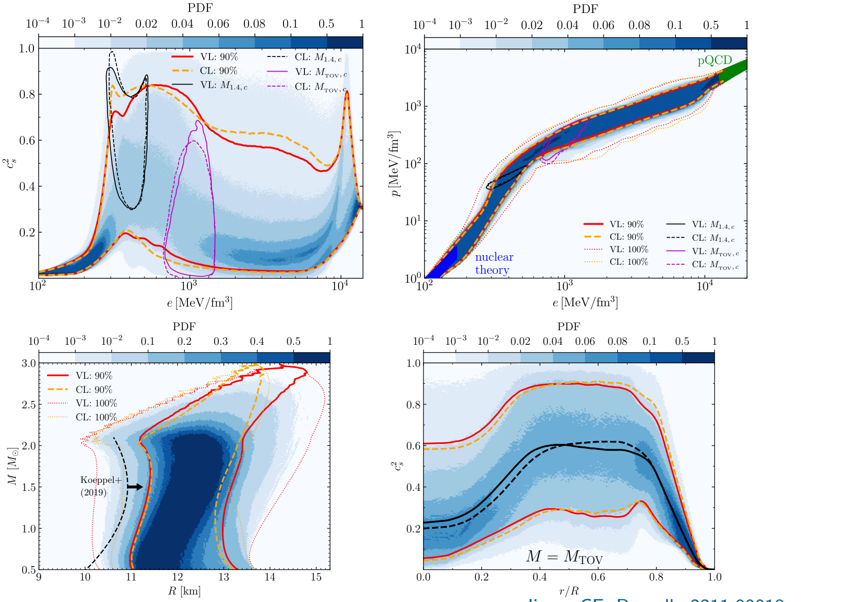
$$c_1 = 0.9008 \quad d_1 = 0.5034 \quad d_2 = 1.452 \quad \nu_1 = 0.3553 \quad \nu_2 = 0.9101\,.$$

Kurkela, Romatschke, Vuorinen arXiv:0912.1856; Fraga, Kurkela, Vuorinen arXiv:1311.5154



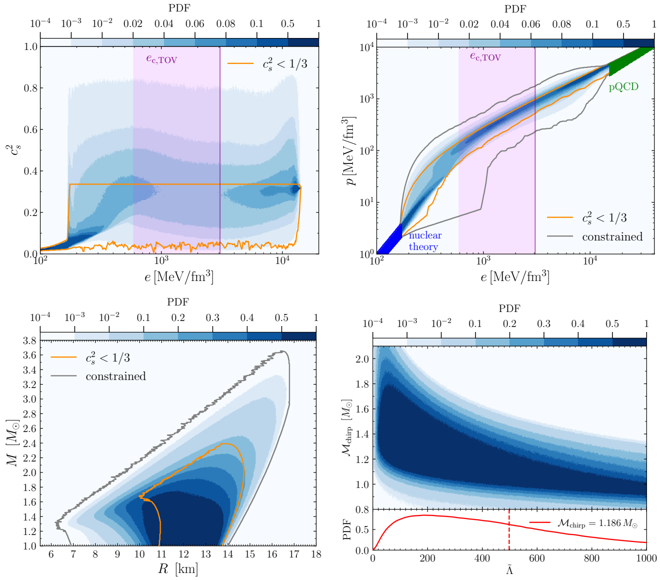



# Bayesian Analysis

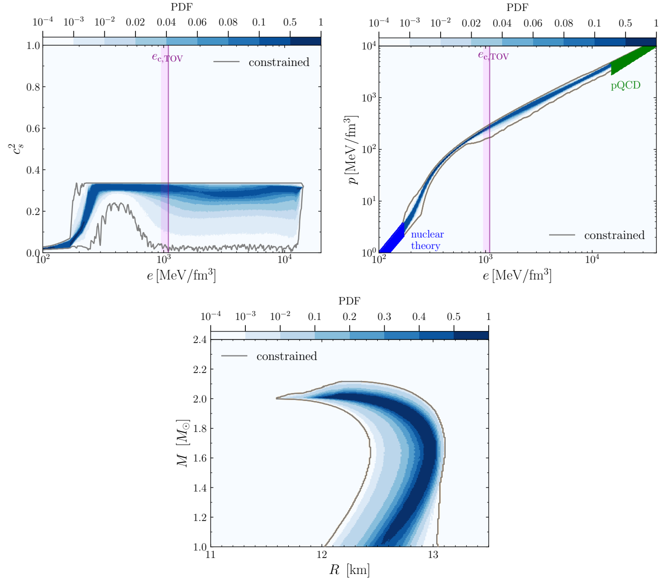



Jiang, CE, Rezzolla 2211.00018

# Without Astro-Constraints



# Sub-Conformal Models



# Different Number of Segments

- Sound speed and MR when using 3,4,5 and 7 segments.
- Features at neutron star densities are robust.