Listening to the long ringdown: a novel way to pinpoint the EOS in neutron-star cores

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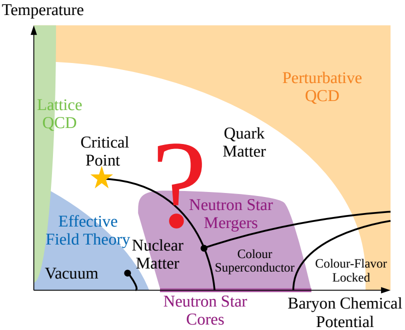
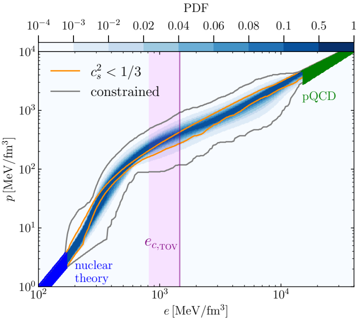
Based on Nature Commun. 16 (2025) 1, 1320 (2403.03246) with
Tyler Gorda, Aleksi Kurkela and Luciano Rezzolla

Motivation

- **Goal:** Infer QCD phase diagram and the equation of state (EOS).
- **Approach:** Synthesis of EOS theory, simulation and observation.
- **This talk:** Using relation between gravitational wave energy and angular momentum (long-ringdown slope) as observable.

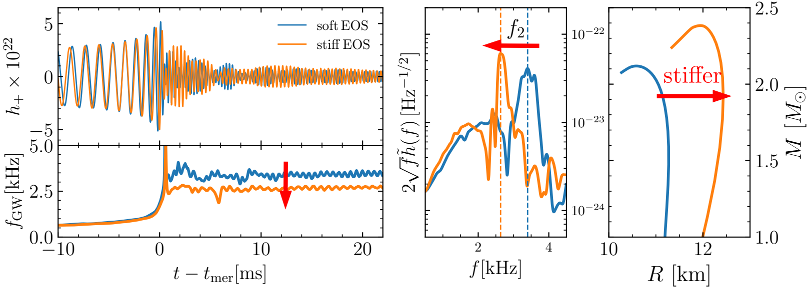




Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Correlating Post-Merger GWs with EOS Properties

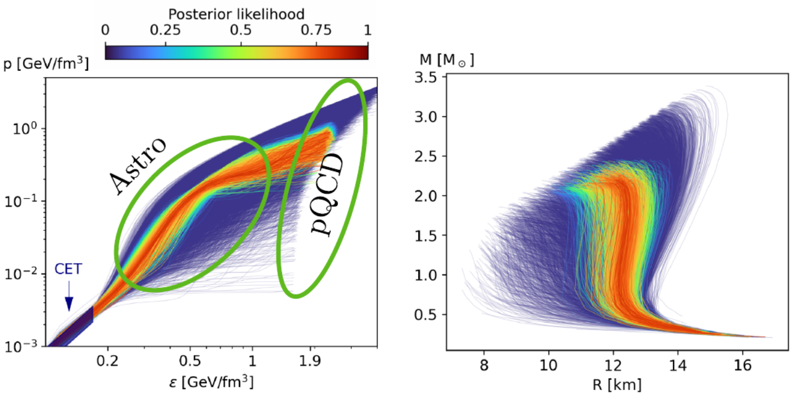
- Spectral features ($f_{\rm GW}$, f_2, etc.) of post-merger GWs correlate with EOS.
  many works: ...; Takami, Rezzolla, Baiotti 1412.3240 (PRD); Bauswein, Stergioulas 1502.03176 (PRD); Rezzolla, Takami 1604.00246 (PRD); De Pietri et al 1910.04036 (PRD); Kiuchi et al 2211.07637 (PRL); Breschi et al 2110.06957 (PRL); ...
- Mechanism: stiff EOS give large radii, slow rotation and low frequencies.
- Not observed so far, but expected to be seen by third-generation detectors: Cosmic Explorer + Einstein Telescope $\approx$ 180 BNSs/year (SNR>8).
  see, e.g., Evans et al 2109.09882 (Cosmic Explorer technical report)



Generic EOS Approach

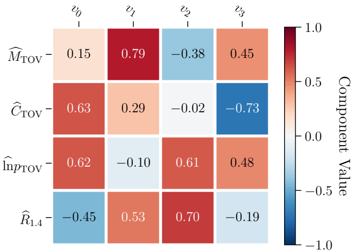
- So far: EOS-GW correlation studies use few "traditional" EOS models.
- However, generic EOS parametrizations reveal large freedom.
- Idea: Large ensemble ($> 10^5$) of generic EOSs that are constrained by astro, nuclear theory and perturbative QCD and cover allowed space.
- EOS ensemble: Gaussian process regression method conditioned with Bayesian likelihoods from dense matter theory and astro measurements.

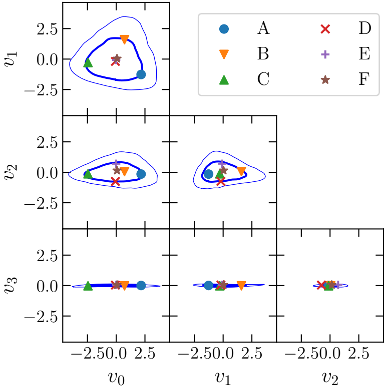
Gorda, Komoltsev, Kurkela 2204.11877 (ApJ)



Golden EOS Selection

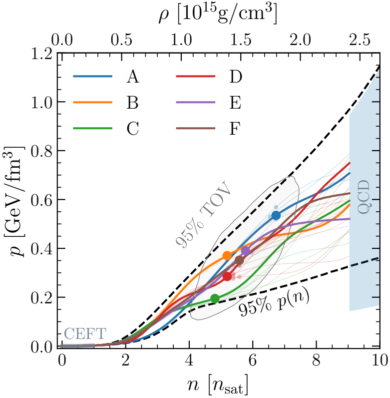
- Too many possibilities to simulate: smart selection recipe is needed.
- Three variables $(M_{\rm TOV}, \mathcal{C}_{\rm TOV}, \ln p_{\rm TOV})$ to characterize the high-density part of the EOS and one $(R_{1.4})$ to break degeneracy at low densities.
- Principle component analysis: 4D distribution essentially 3D-triangular.
- Six "golden EOSs": A-E at corners of 68% contours and F in centre.

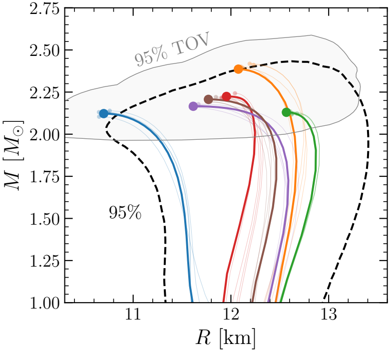




Golden EOSs and Mass-Radius Relations

- Six EOS models manageable, but BNS parameter space still huge: fix $\mathcal{M}_{\rm chirp} = 1.18\ M_\odot$, but three different mass-ratios $q = \frac{M_1}{M_2} = 1, 0.85, 0.7$.
- Add T-dependence via simple gamma-law approximation with fixed $\Gamma_{\rm th} = 1.75$, but analysis remains robust when changing to $\Gamma_{\rm th} = 1.5, 2.0$.





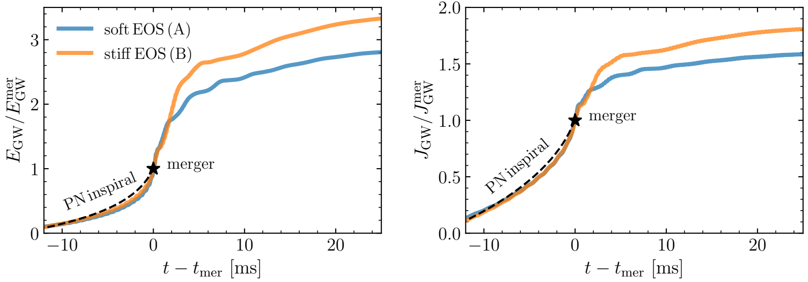
Emitted GW Energy and Angular Momentum

- Time evolution of GW strain components $h_+, h_\times$ from simulation.
- Emitted energy and angular momentum from post-processing

$$E_{\mathrm{GW}}(t) = \frac{r^2}{16\pi}\int_{-\infty}^{t} dt' \left(\dot{h}_+^2 + \dot{h}_\times^2\right), \; J_{\mathrm{GW}}(t) = \frac{r^2}{16\pi}\int_{-\infty}^{t} dt' \left(h_+ \dot{h}_\times - \dot{h}_+ h_\times\right).$$

see, e.g., Bishop, Rezzolla 1606.02532 (Living Reviews in Relativity)

- Useful to normalise with $E_{\mathrm{GW}}^{\mathrm{mer}} := E_{\mathrm{GW}}(t_{\mathrm{mer}})$ and $J_{\mathrm{GW}}^{\mathrm{mer}} := J_{\mathrm{GW}}(t_{\mathrm{mer}})$.



5th-order post-Newtonian Taylor-T2 mode with $\tilde{\Lambda} = 580$ of PyCBC library, Biwer et al 1807.10312 (Publ.Astron.Soc.Pac.)

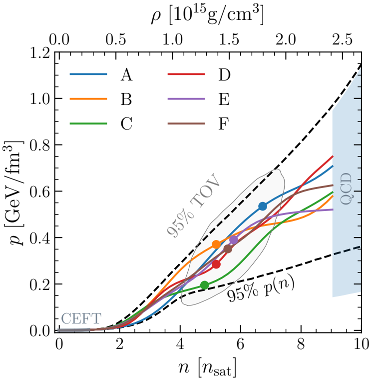
Long Ringdown

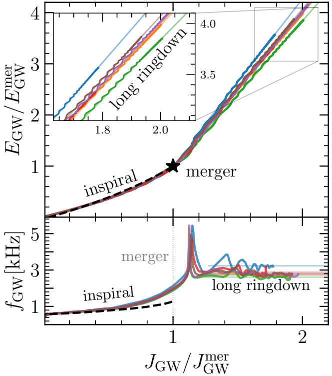
- Post-merger period where $E_{\rm GW}(t)$ and $J_{\rm GW}(t)$ are linearly related.
- Long-ringdown slope numerically close to GW frequency

$$\frac{dE_{\rm GW}}{dJ_{\rm GW}} = \frac{\dot{E}_{\rm GW}}{\dot{J}_{\rm GW}} = \frac{\dot{h}_+^2 + \dot{h}_\times^2}{h_+\dot{h}_\times - \dot{h}_+ h_\times}\,, \quad f_{\rm GW} = \frac{1}{2\pi}\,\frac{h_+\dot{h}_\times - \dot{h}_+ h_\times}{h_+^2 + h_\times^2}\,.$$

- Identity for simple quadrupole system with $\ell = 2, m = 2$ deformation:

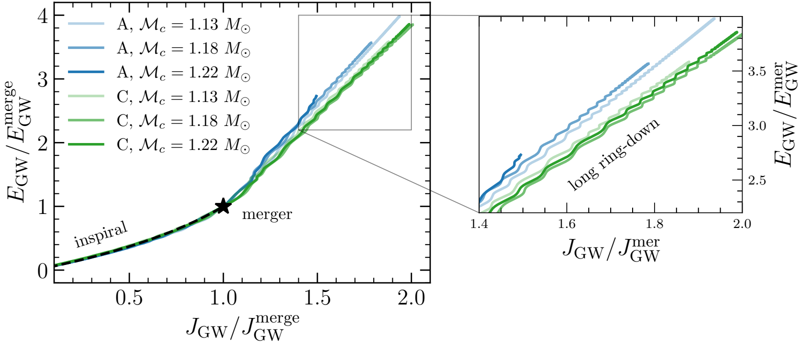
$$h_+(t) \propto \cos(\phi(t))\,, \quad h_\times(t) \propto \sin(\phi(t))\,, \quad \dot{E}_{\rm GW}/\dot{J}_{\rm GW} = f_{\rm GW}/(2\pi)\,.$$





Robustness

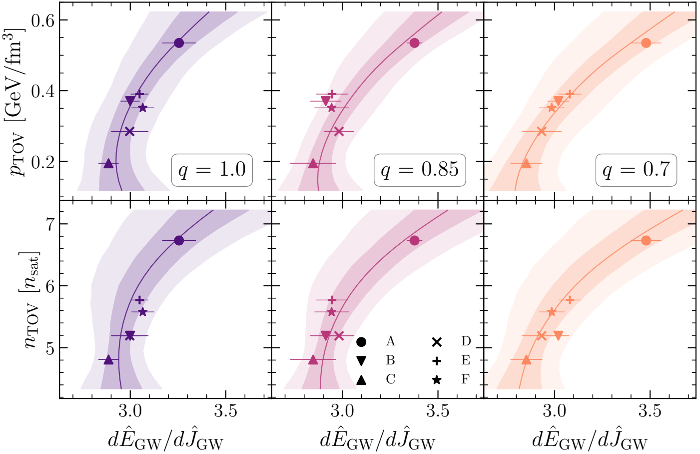
- Waveform dominated by the large-scale deformations of post-merger remnant, only weak influence of small-scale features.
- Essentially no difference between simulations with $200 - 400$ m resolution.
- Also insensitive to approximate thermal effects for $\Gamma_{\rm th} = 1.5 - 2$.
- However, sensitive to BNS parameters such as chirp mass, but sufficiently different EOSs remain distinguishable.



Correlations

- Slope correlates with maximum neutron star pressure and number density.
- Bilinear fit of simulation data constrains EOS at highest (TOV) densities

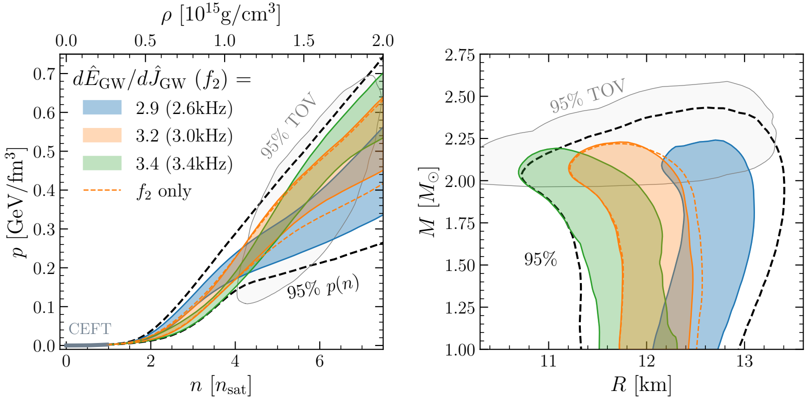
$$\frac{d\hat{E}_{\mathrm{GW}}}{d\hat{J}_{\mathrm{GW}}} = \beta_0 + \beta_1\, p_{\mathrm{TOV}} + \beta_2\, n_{\mathrm{TOV}} + \beta_3\, q + \beta_4\, q\, p_{\mathrm{TOV}} + \beta_5\, q\, n_{\mathrm{TOV}} + \beta_6\, p_{\mathrm{TOV}}\, n_{\mathrm{TOV}}\,.$$



Impact of Slope Measurement

- Mock measurement: assume measured values for the slope $d\hat{E}_{\mathrm{GW}}/d\hat{J}_{\mathrm{GW}}$ and f_2 with $\pm 4\%$ measurement uncertainty.
- Correlation gives new evidence to update EOS constraints at all densities.
- Slightly improvement compared to measuring just f_2.
- Pearson-correlation coefficients slightly larger for $d\hat{E}_{\mathrm{GW}}/d\hat{J}_{\mathrm{GW}}$ than for f_2

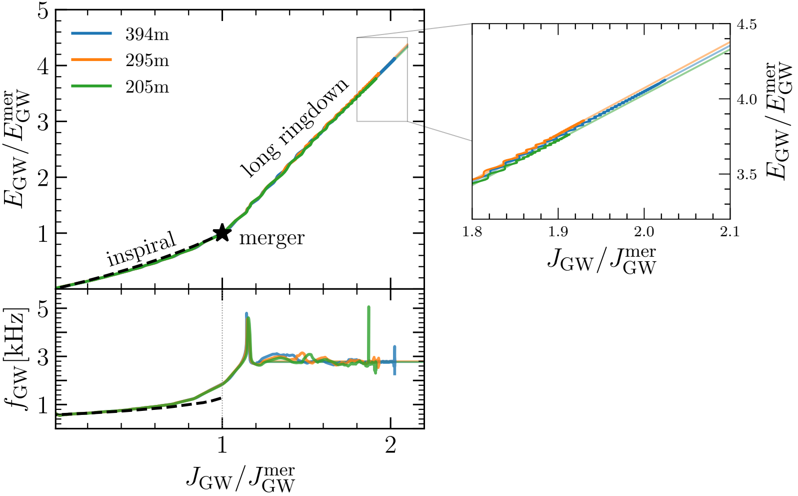
$$r(dE_{\mathrm{GW}}/dJ_{\mathrm{GW}}, p_{\mathrm{TOV}}) = 0.877 \quad \text{vs} \quad r(f_2, p_{\mathrm{TOV}}) = 0.792\,.$$



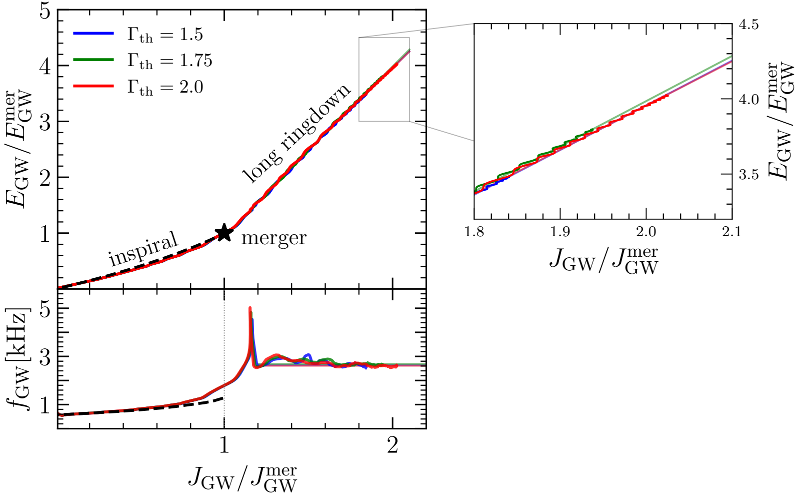
Summary

- Third-generation GW observatories are expected to see numerous BNS post-merger waveforms at high SNR.
- Looking for EOS-GW correlations using large generic EOS set.
- Principle component analysis to single out few golden EOSs.
- Novel correlations between long ringdown slope $d\hat{E}_{\rm GW}/d\hat{J}_{\rm GW}$ and $(p_{\rm TOV}, n_{\rm TOV})$ constrain the EOS at maximum NS core densities.
- Slope $d\hat{E}_{\rm GW}/d\hat{J}_{\rm GW}$ simple to compute from the waveform.
- Correlation + Bayes theorem: EOS (MR) constraints at all densities.
- Improvement compared to measuring f_2 only.

Impact of Resolution



Impact of Gamma



Tabulated Models