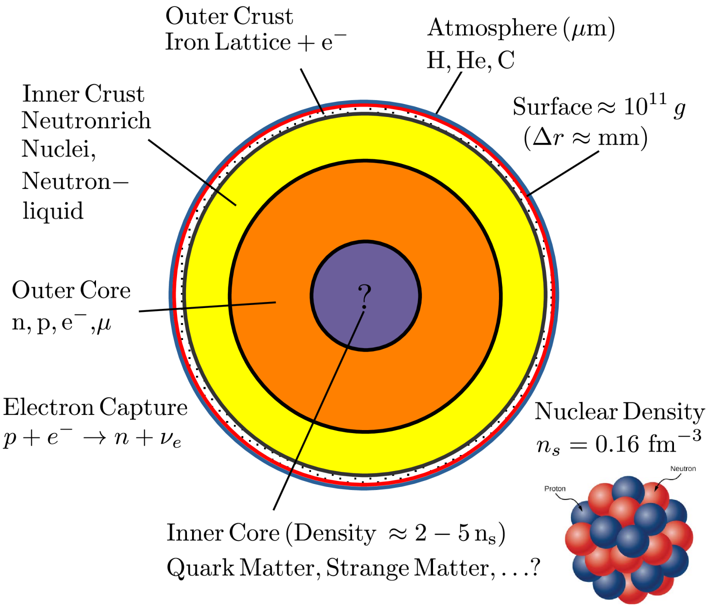
On the Sound Speed in Neutron Stars

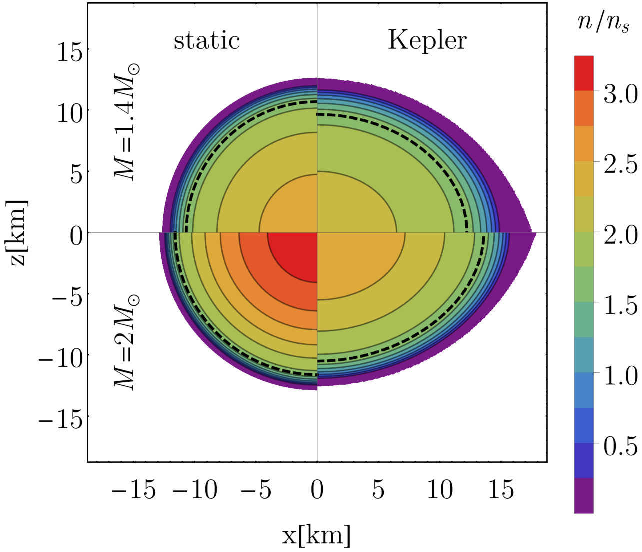
Christian Ecker

From Holography to Machine Learning
Helsinki, 25 October 2022

Based on 2203.14974 (ApJL), 2207.04417 (ApJL) and 2209.08101 with
Sinan Altiparmak and Luciano Rezzolla


Outer Crust
Iron Lattice + e^-
Atmosphere (μm)
H, He, C
Inner Crust
Neutronrich
Nuclei,
Neutron–
liquid
Surface $\approx 10^{11}\ g$
($\Delta r \approx$ mm)
?
Outer Core
n, p, e^-, μ
Electron Capture
$p + e^- \rightarrow n + \nu_e$
Nuclear Density
$n_s = 0.16\ \mathrm{fm}^{-3}$
Neutron
Proton
Inner Core (Density $\approx 2 - 5\,\mathrm{n_s}$)
Quark Matter, Strange Matter, . . . ?

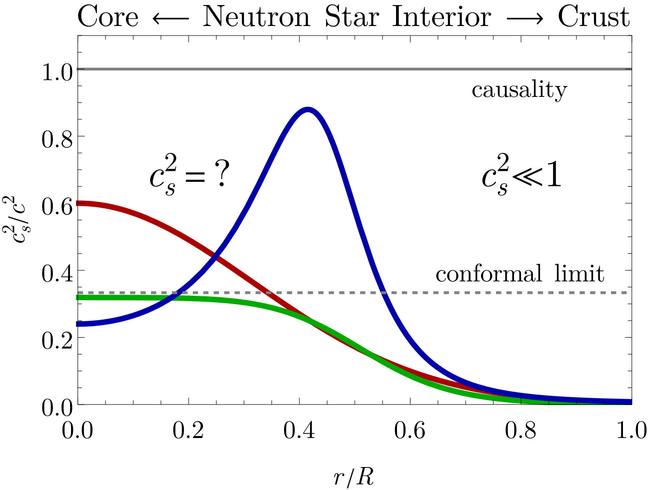
Density Profile



Demircik, CE, Järvinen 2009.10731 (ApJL)

Speed of Sound

$$c_s^2 = \left(\frac{dp}{de}\right)_s \,, \quad 0 \leqslant c_s^2 \leqslant 1\,.$$



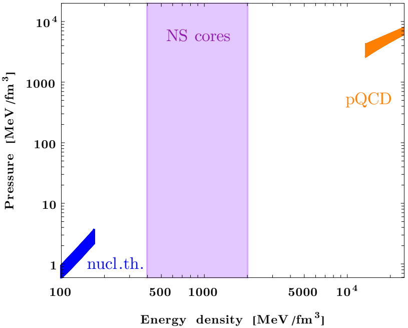
Equation of State

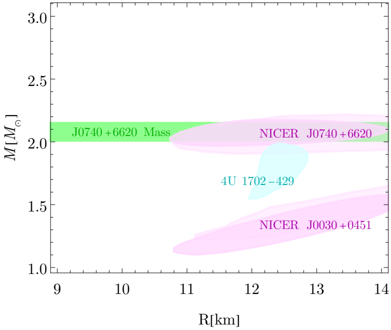
Relates properties of dense matter to properties of neutron stars.

$$p = p(e, \dots) \xrightarrow{\text{Einstein Eq.}} M(R)$$

Constrained by theory and neutron star observations:

- Causality & thermodynamic stability: $0 < c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} < 1$
- Direct mass and radius measurements: $M_{\mathrm{TOV}} \gtrsim 2\,M_\odot$, $11\,\mathrm{km} \lesssim R \lesssim 14\,\mathrm{km}$
- First GW detection from a BNS merger GW170817: $\tilde{\Lambda} \lesssim 720$





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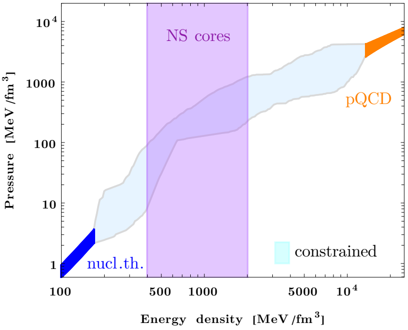
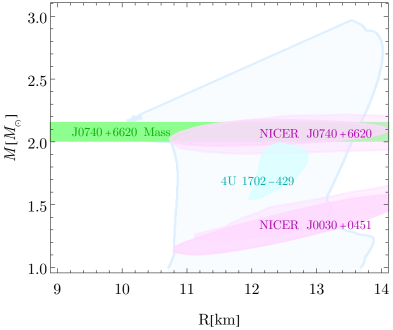


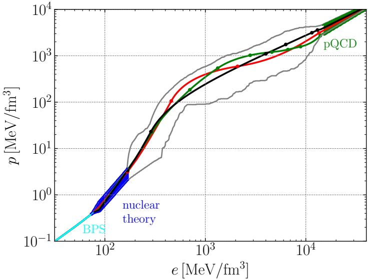


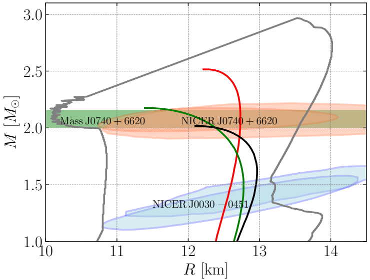
Agnostic Approach

- Impose boundary conditions from nuclear theory and perturbative QCD.
  Hebeler, Lattimer, Pethick, Schwenk 1303.4662 (ApJ);
  Fraga, Kurkela, Vuorinen 1311.5154 (ApJL)
- Connect with random, but causal and thermodynamically consistent EOS.

$$c_s^2(\mu) = \frac{(\mu_{i+1}-\mu)c_{s,i}^2 + (\mu-\mu_i)c_{s,i+1}^2}{\mu_{i+1}-\mu_i}\,,\; n(\mu) = n_1\, e^{\int_{\mu_1}^{\mu} \frac{\mathrm{d}\mu'}{\mu' c_s^2(\mu')}}\,,\; p(\mu) = p_1 + \int_{\mu_1}^{\mu} \mathrm{d}\mu'\, n(\mu')\,.$$

  Annala, Gorda, Kurkela, Nättilä, Vuorinen 1903.09121 (Nature Physics)
- Constrain with pulsar and binary neutron star observations.





Lots of recent work:
...; Annala, Gorda, Kurkela, Nättilä, Vuorinen 2105.05132 (PRX);
Komoltsev, Kurkela 2111.05350 (PRL); Altiparmak, CE, Rezzolla (ApJL) 2203.14974;
Gorda, Komoltsev, Kurkela 2204.11877; CE, Rezzolla 2207.04417 (ApJL); ...

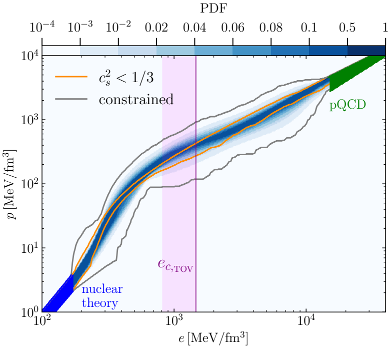
Sampling the EOS

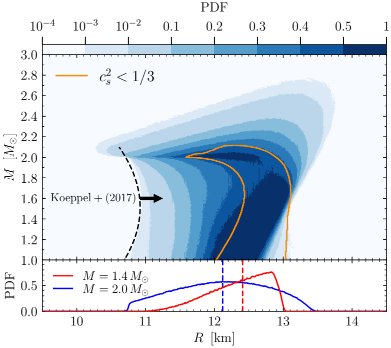
- Statistics of millions of EOS models that pass theory+astro constraints.
- Mass bound of generic (sub-conformal) models: $M_{\rm TOV} < 3\,(2.1)\,M_\odot$.
- Radius estimates (+ error bars) for typical stars:

$$R_{1.4} = 12.42^{+0.52}_{-0.99}\,\mathrm{km}\,,\ R_{2.0} = 12.11^{+1.11}_{-1.23}\,\mathrm{km}\,(95\%\,\mathrm{confidence})\,.$$

- Remarkable agreement with $R_{min}(M)$ from threshold mass calculations.

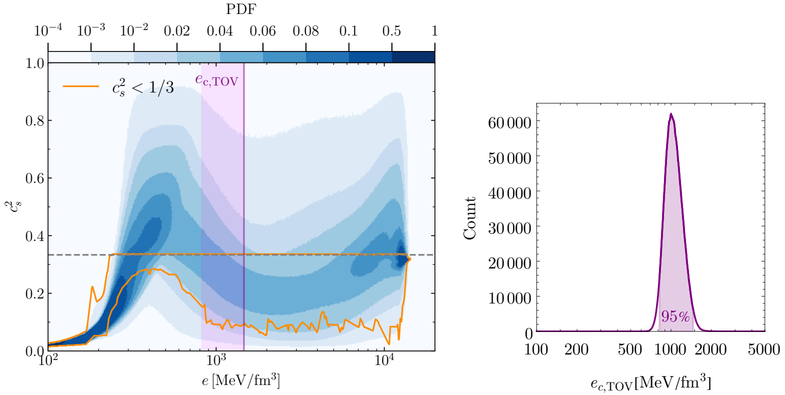
Koeppel, Rezzolla 1901.09977 (ApJL)





Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Sound Speed in Dense Matter



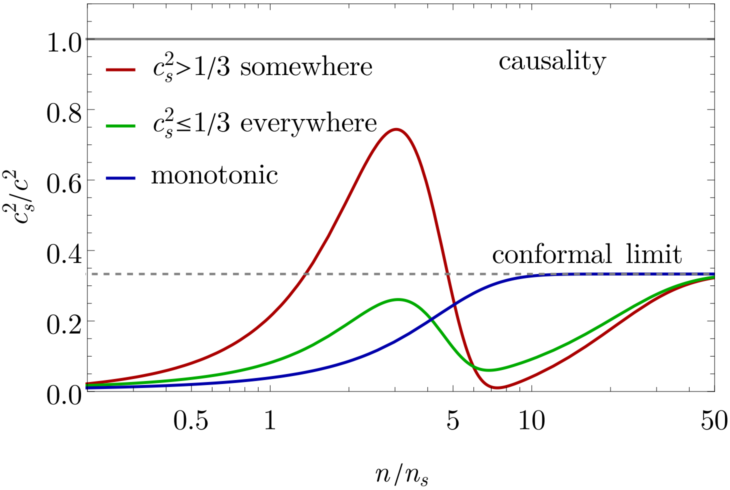
- Local maximum $c_s^2 > 1/3$ below maximal central densities of TOV stars

$$e_{c,\mathrm{TOV}} = 1064^{+399}_{-244}\,\mathrm{MeV/fm^3}\ (95\%\ \mathrm{confidence})\,.$$

- $c_s^2 \leqslant 1/3$ at all densities possible, but only a small fraction (0.03%).
- Lower bound $c_s^2 \gtrsim 0.2$ of sub-conformal models around $e \approx 400\,\mathrm{MeV/fm^3}$.

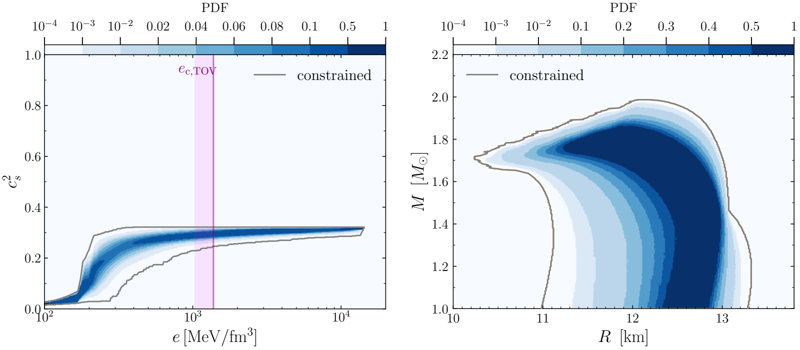
Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Several Different Possibilities



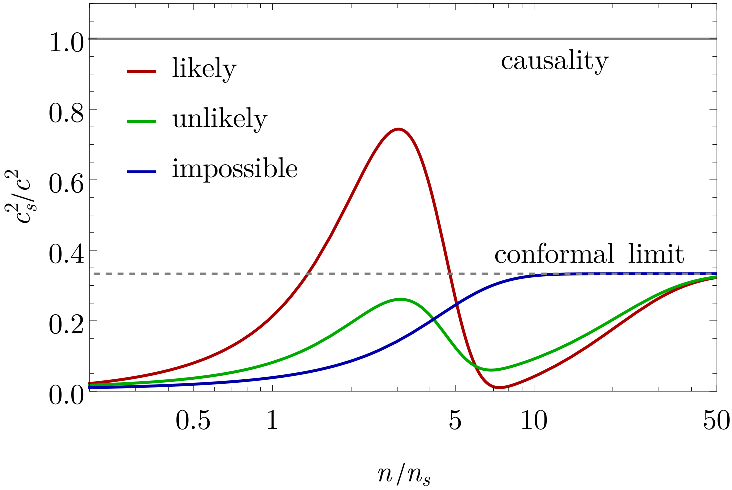
Monotonic Sound Speed

- Separate calculation with more than 10^7 models.
- Monotonic by construction, but no lower mass bound.
- Can only reach $M_{TOV} < 2\,M_\odot$.

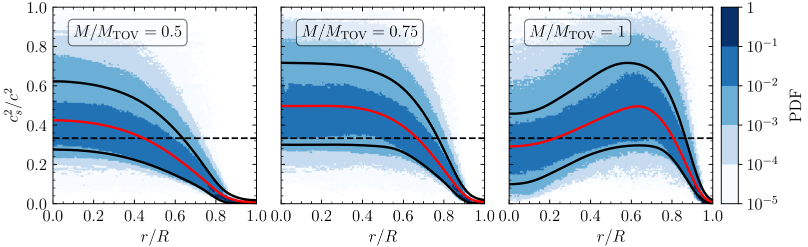


Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Fewer Possibilities



Sound Speed Inside Neutron Stars

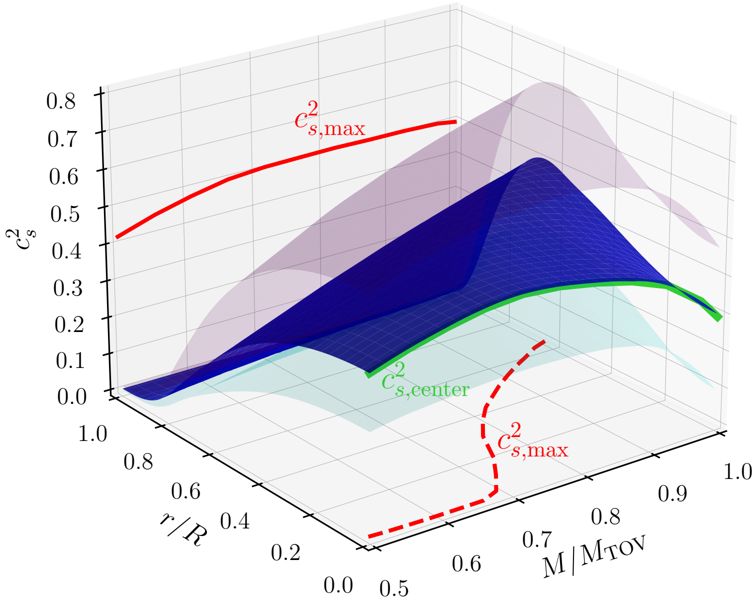


Median (red) and 95% intervals (black) can be accurately described by

$$c_s^2(x) = \left(\alpha e^{\beta x^2} + \gamma e^{\delta (x-\epsilon)^2}\right)\left[1 - \tanh\left(\zeta x - \eta\right)\right], \quad x \equiv r/R\,.$$

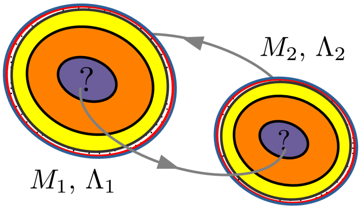
- **Light Stars:** stiff core, soft outer layer.
- **Typical Stars:** large stiff core.
- **Massive Stars:** soft core, stiff outer layer.

CE, Rezzolla 2207.04417 (ApJL)



CE, Rezzolla 2207.04417 (ApJL)

Chirp Mass and Binary Tidal Deformability



- Chirp mass can be extracted accurately from inspiral GW strain frequency

$$\mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = \left(\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right)^{3/5} .$$

- Binary tidal deformability encodes EOS properties via Λ_i

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12 M_2 + M_1) M_1^4 \Lambda_1 + (12 M_1 + M_2) M_2^4 \Lambda_2}{(M_1 + M_2)^5}, \quad \Lambda_i = \tfrac{2}{3} k_2 (R_i/M_i)^5 .$$

- GW170817 only "good" event so far:

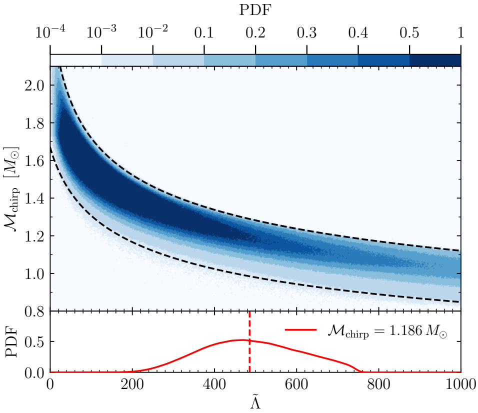
$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002} M_\odot, \quad q = M_2/M_1 > 0.7, \quad \tilde{\Lambda} \lesssim 720 .$$

Binary Tidal Deformability

- More than 10^8 binary configurations with $M_{1,2} \in [0.5M_\odot, M_{\rm TOV}]$.

$$\tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}^c_{chirp} \longrightarrow \tilde{\Lambda}_{1.186} = 485^{+225}_{-211}\,,$$

$$a = -50(-20),\ b = 500(1800),\ c = -4.5(-5.0)\,.$$



Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

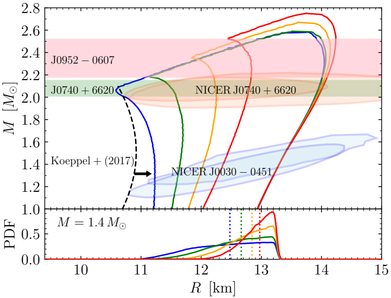
see also: Zhao, Lattimer 1808.02858 (PRD)

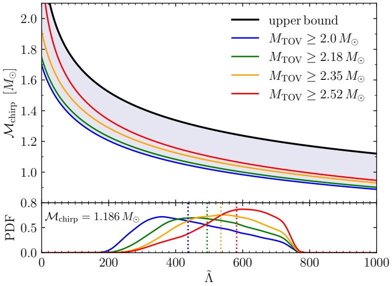
Black-Widow Binary Pulsar PSR J0952-0607

- Most massive NS known: $M = 2.35 \pm 0.17\,M_\odot$, but large uncertainties!

  Romani, Kandel, Filippenko, Brink, Zheng 2207.05124 (ApJL)

  also: PSR J2215+5135 ($M = 2.27^{+0.17}_{-0.15}\,M_\odot$) Linares, Shahbaz, Casares 1805.08799 (ApJ)
- Large mass constraints dominate over radius constraints by NICER.





$M_{\rm TOV}$ $[M_\odot]$	a	b	c
$\geqslant 2.00$	-50	600	4.7
$\geqslant 2.18$	-45	650	4.6
$\geqslant 2.35$	-40	750	4.5
$\geqslant 2.52$	-20	800	4.4

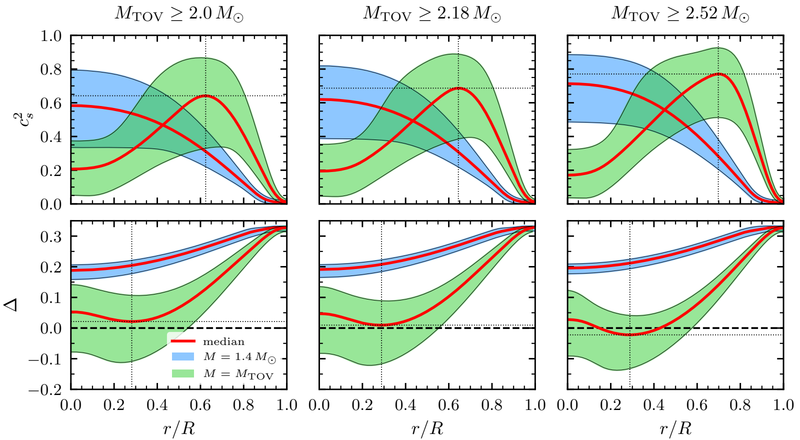
CE, Rezzolla 2209.08101

Sound Speed and Conformal Anomaly

$$\Delta := \frac{1}{3}\frac{g_{\mu\nu}T^{\mu\nu}}{e} = \frac{1}{3} - \frac{p}{e}\,, \quad -\frac{2}{3} \leqslant \Delta \leqslant \frac{1}{3}\,.$$

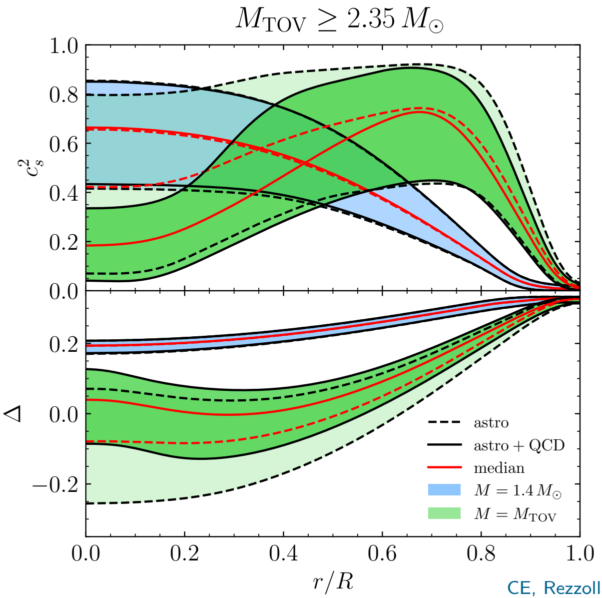
Fujimoto, Fukushima, McLerran, Praszalowicz 2207.06753

Marczenko, McLerran, Redlich, Sasaki 2207.13059



CE, Rezzolla 2209.08101

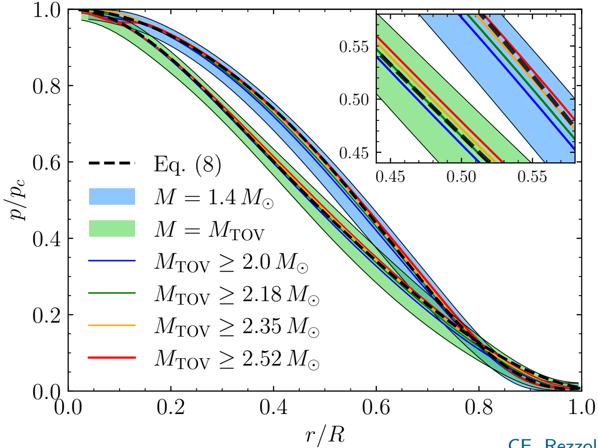
Impact of QCD



CE, Rezzolla 2209.08101

Quasi-Universality of the Pressure Profile

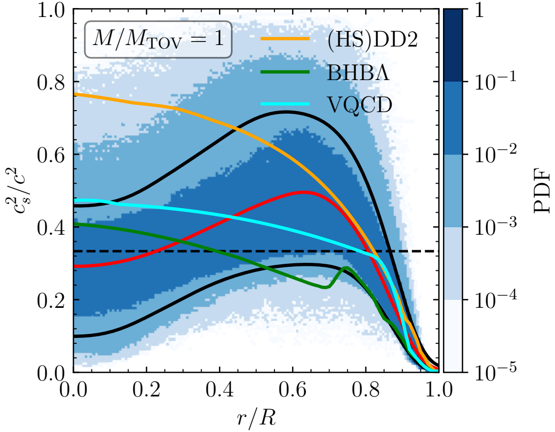
$$\frac{p(x)}{p_c} = e^{\alpha x^2} + e^{\alpha}\,\frac{\cos\beta x - 1}{\cos\beta - 1}\,, \quad x \equiv r/R\,.$$



CE, Rezzolla 2209.08101

Puzzle

- *Is there a micro-physical model that can explain the agnostic sound-speed profile inside neutron stars?*



CE, Rezzolla 2207.04417 (ApJL)

Summary

This talk:

- Basic properties (+ error-bars) of the cold EOS and neutron stars can be estimated from constraint sampling: $p(e)\,, M(R)\,, c_s^2\,, \tilde{\Lambda}(\mathcal{M}_{\rm chirp})\,, \ldots$
- Using r/R allows to sample the NS interior: $c_s^2(r/R),\ \Delta(r/R),\ \ldots$
- Light stars have a stiff core and soft outer layers, massive stars have soft cores and stiff outer layers.
- Can provide additional constraints for micro-physical EOS model building and help to identify quasi-universal relations.

Ongoing work:

- Comparison of full Bayesian analysis to simple cutoff approach (this talk).

  Yiang, CE, Rezzolla (online soon)

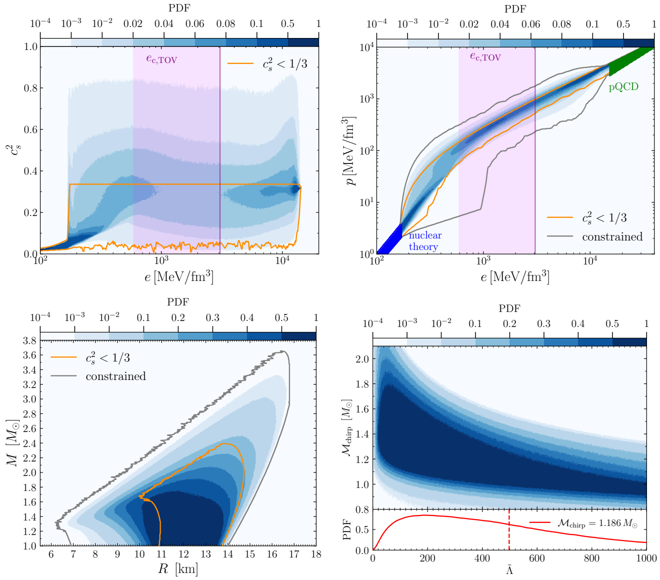
- Model specific sampling: narrow V-QCD parameter space with astro, ...

  CE, Jokela, Järvinen (in progress)

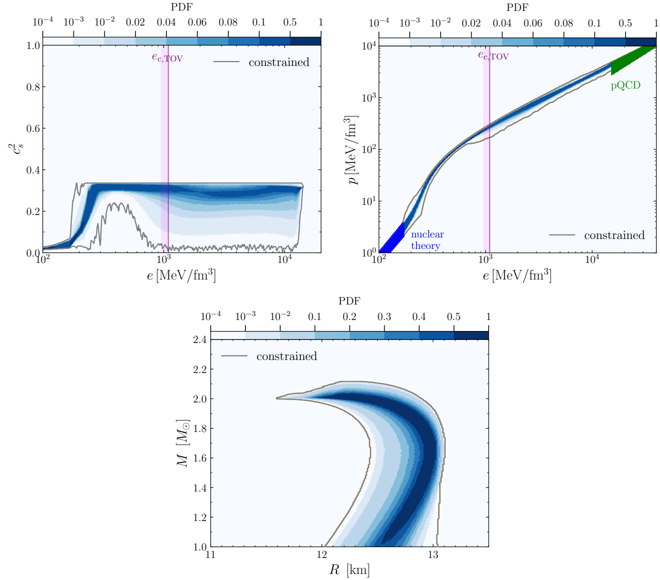
- Sampling BNS-observables: threshold mass, post-merger frequencies, etc.

  CE, Gorda, Rezzolla (in progress)

Without Astro-Constraints



Sub-Conformal Models



Different Number of Segments

- Sound speed and MR when using 3,4,5 and 7 segments.
- Features at neutron star densities are robust.