Exploring the Phase Diagram of QCD with Neutron Star Mergers in the Prompt and Non-Prompt Collapse Regime

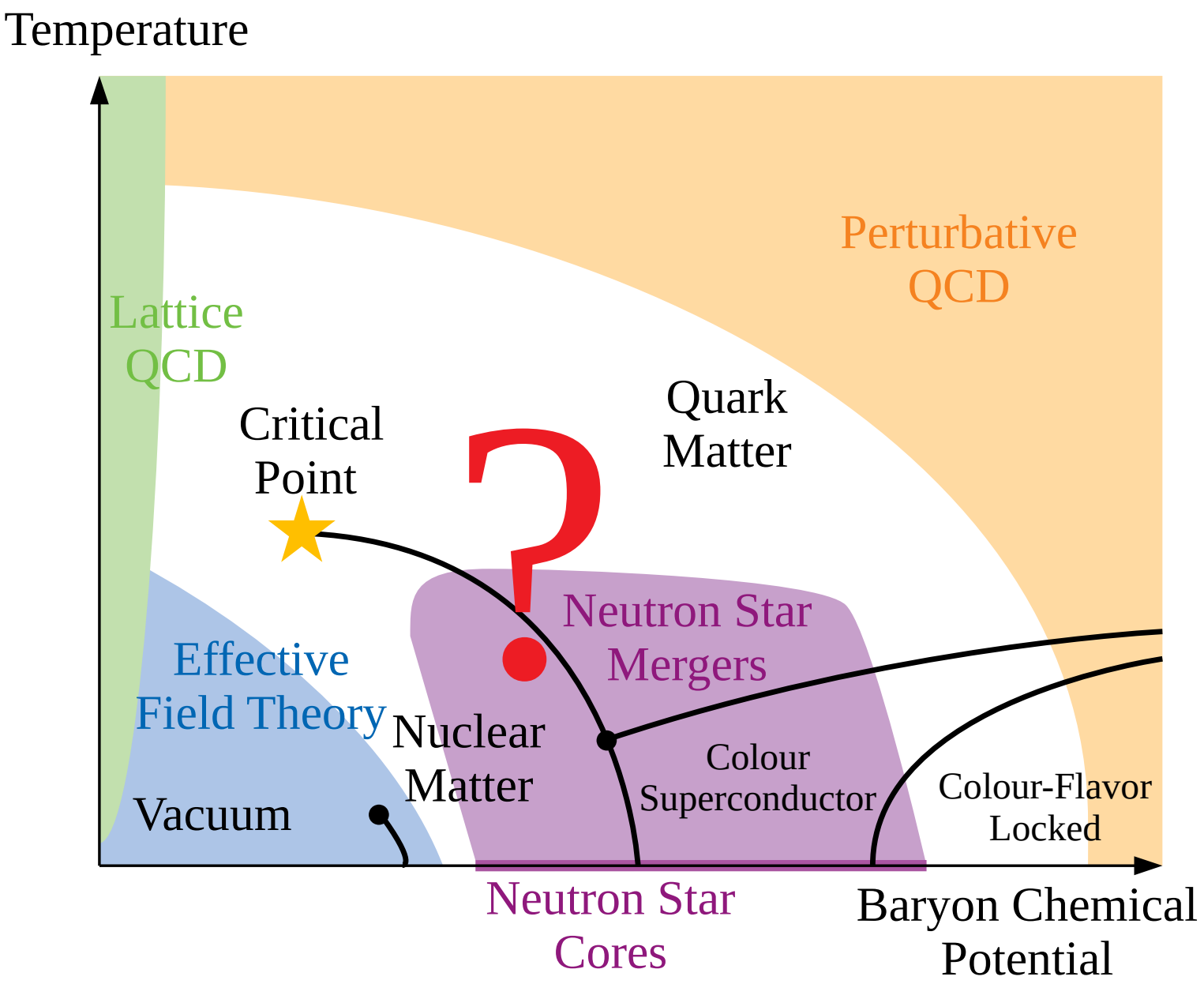
Christian Ecker

HIP-Seminar, 11 April 2024

Based on recent work 2402.11013 with
Matti Järvinen, Konrad Topolski and Alina Stehr

and older works 2112.12157 (PRX), 2205.05691 (SciPost), 2203.14974 (ApJL), 2207.04417 (ApJL), 2209.08101 (MNRAS) with
Altiparmak, Demircik, Järvinen, Rezzolla, Tootle and Topolski

What we (don't) know about QCD



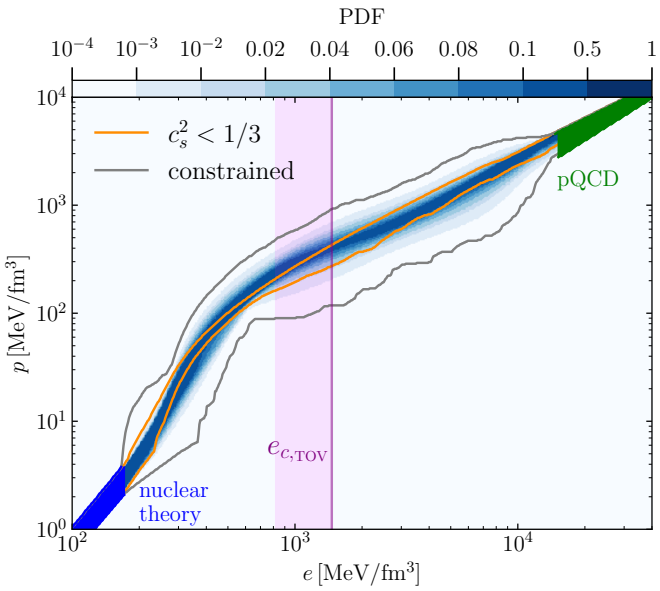
Equation of State

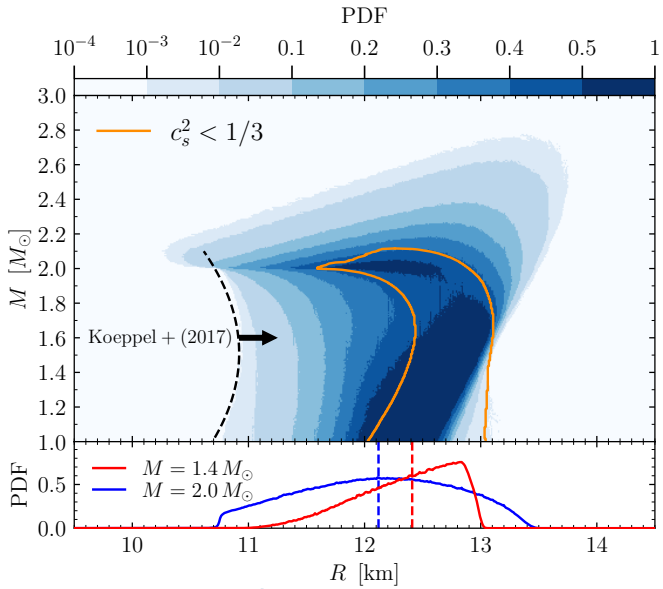
Relates properties of dense matter to properties of neutron stars.

$$p = p(e, \ldots) \xrightarrow{\text{Einstein Eq.}} M(R)$$

Constrained by theory and neutron star observations:

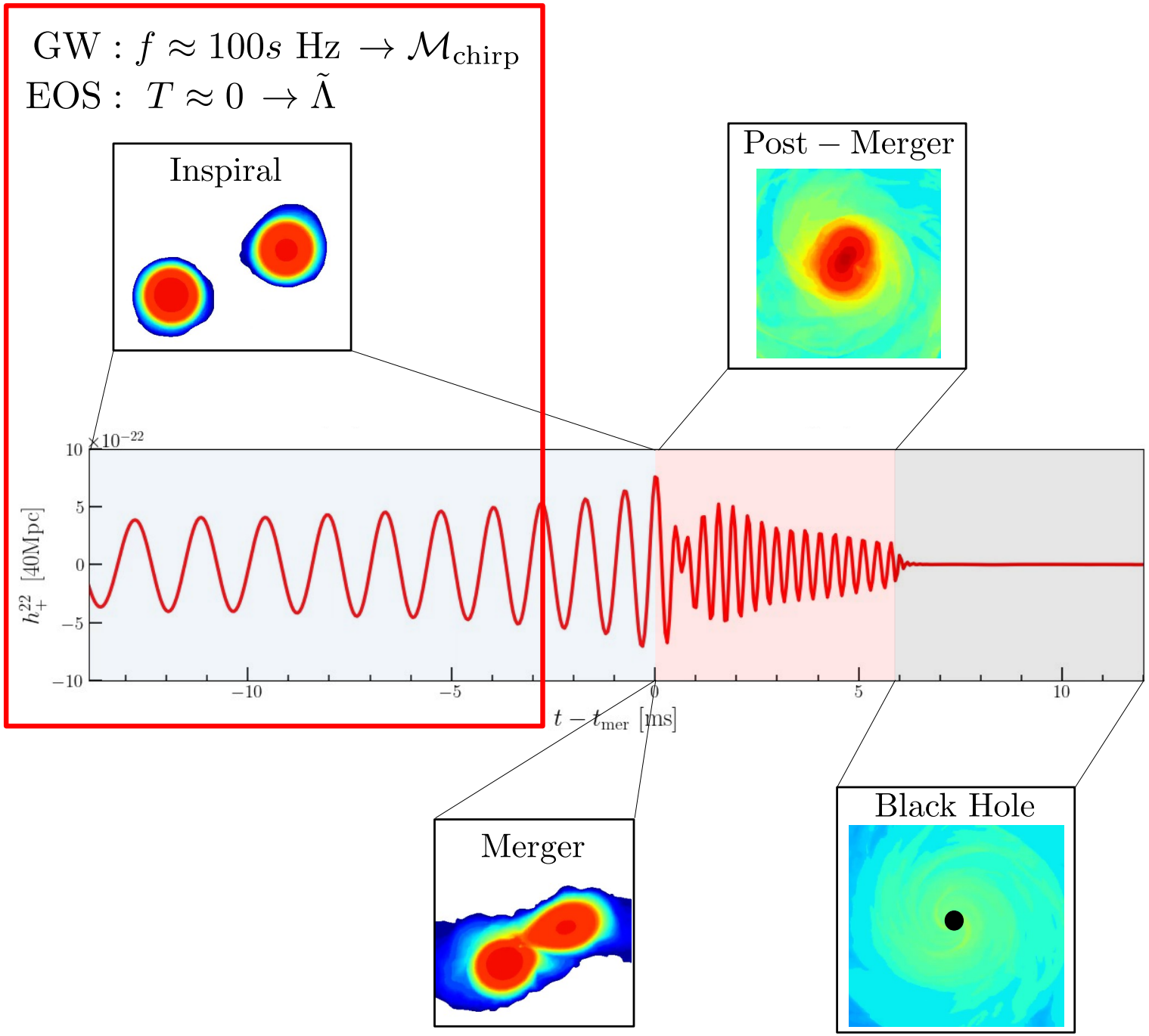
- Causality & thermodynamic stability: $0 < c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} < 1$
- Direct mass and radius measurements: $M_{\mathrm{TOV}} \gtrsim 2\,M_\odot$, $11\,\mathrm{km} \lesssim R \lesssim 14\,\mathrm{km}$
- First GW detection from a BNS merger GW170817: $\tilde{\Lambda} \lesssim 720$



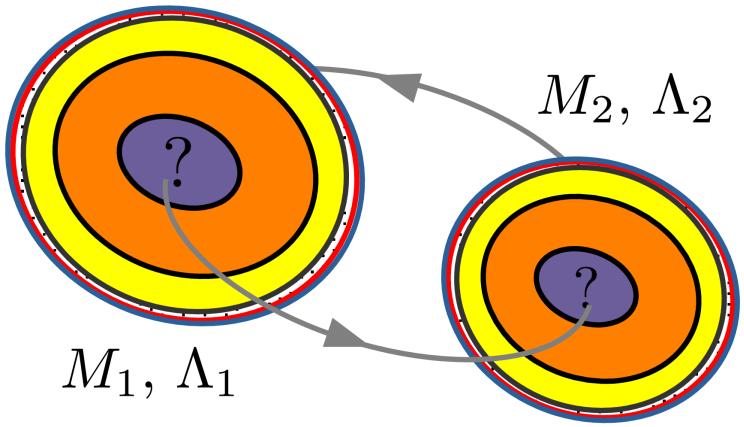


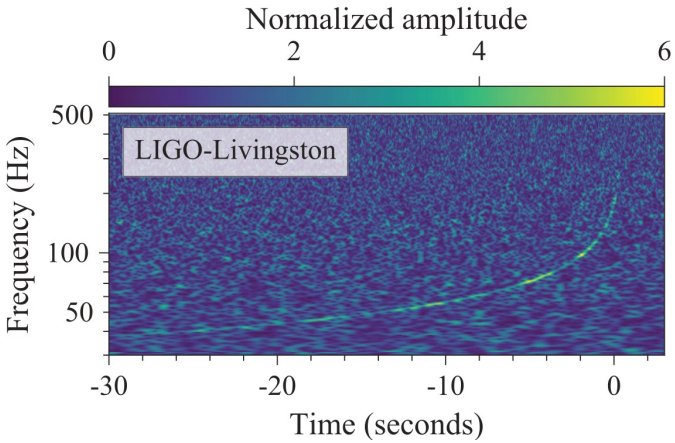
Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Lots of recent work: ...; Annala, Gorda, Kurkela, Nättilä, Vuorinen 2105.05132 (PRX); Komoltsev, Kurkela 2111.05350 (PRL); Altiparmak, CE, Rezzolla (ApJL) 2203.14974; Gorda, Komoltsev, Kurkela 2204.11877; CE, Rezzolla 2207.04417 (ApJL); ...


GW : $f \approx 100s$ Hz $\rightarrow \mathcal{M}_{\rm chirp}$
EOS : $T \approx 0 \rightarrow \tilde{\Lambda}$
Inspiral
Post − Merger
Merger
Black Hole
$\times 10^{-22}$
h_+^{22} [40Mpc]
$t - t_{\rm mer}$ [ms]

Chirp Mass and Binary Tidal Deformability





- Chirp mass can be extracted accurately from inspiral GW strain frequency

$$\frac{\mathrm{d}f_{GW}}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\mathcal{M}_{chirp}^{5/3} f_{GW}^{11/3}\,, \quad \mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}\,.$$

Cutler, Flanagan arXiv:gr-qc/9402014 (PRD)

- Binary tidal deformability encodes EOS properties via Λ_i

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12M_2+M_1)\,M_1^4\Lambda_1 + (12M_1+M_2)\,M_2^4\Lambda_2}{(M_1+M_2)^5}\,, \quad \Lambda_i = \tfrac{2}{3}k_2\,(R_i/M_i)^5\,.$$

- GW170817 "best" event so far:

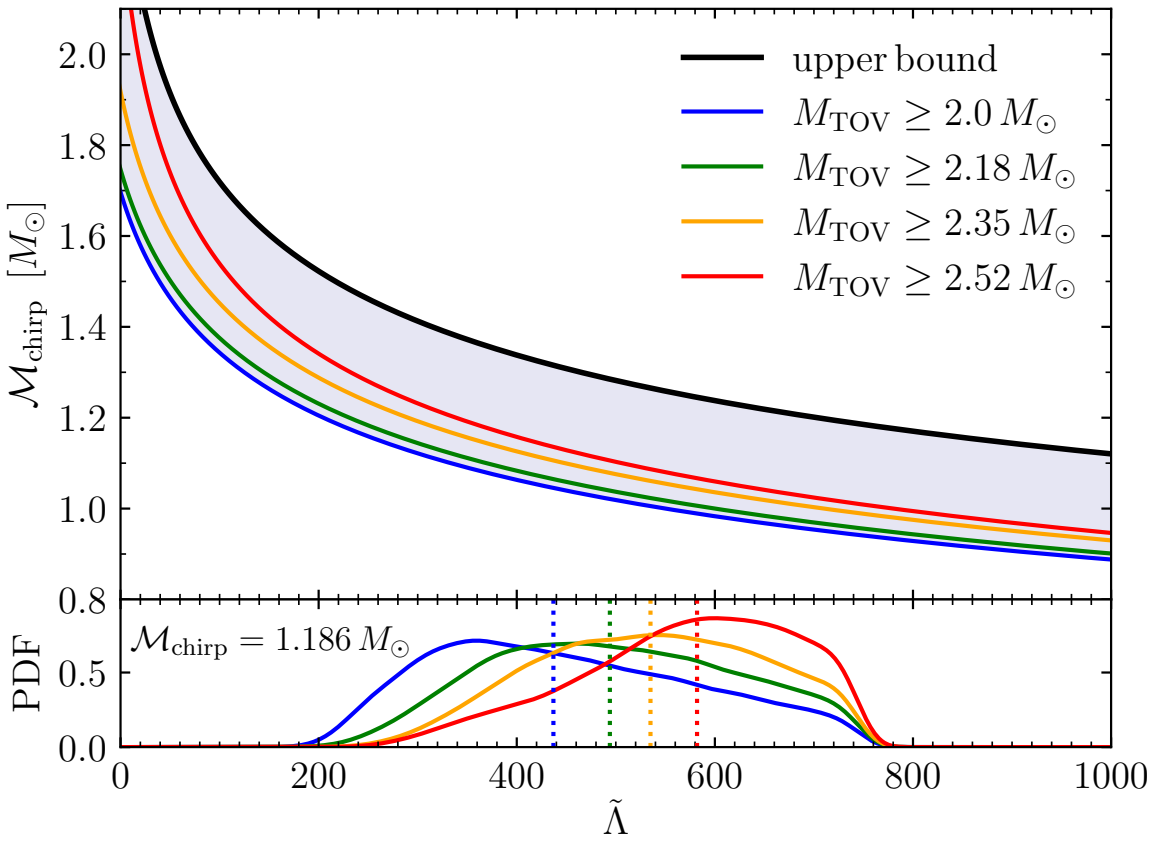
$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002}M_\odot, \quad q = M_2/M_1 > 0.7, \quad \tilde{\Lambda} \lesssim 720\,.$$

LIGO/Virgo: arXiv:1805.11579

Binary Tidal Deformability

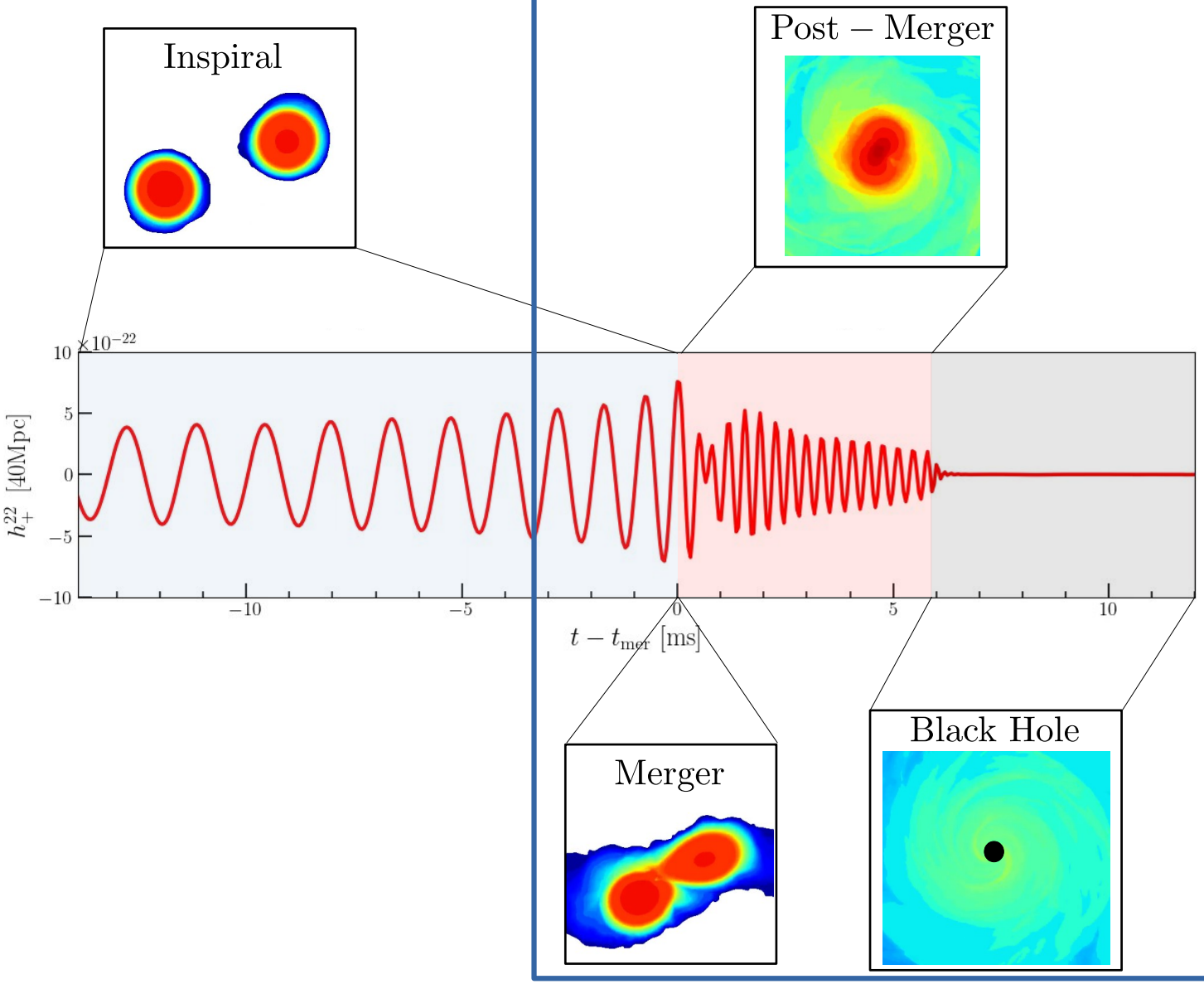
More than 10^8 binary configurations with $M_{1,2} \in [0.5 M_\odot, M_{\rm TOV}]$.

$$\tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}^c_{chirp} \longrightarrow \tilde{\Lambda}^{\rm min}_{1.186} \in [236, 301]\,,\; M_{\rm TOV} > [2.18, 2.52]\; M_\odot\,.$$

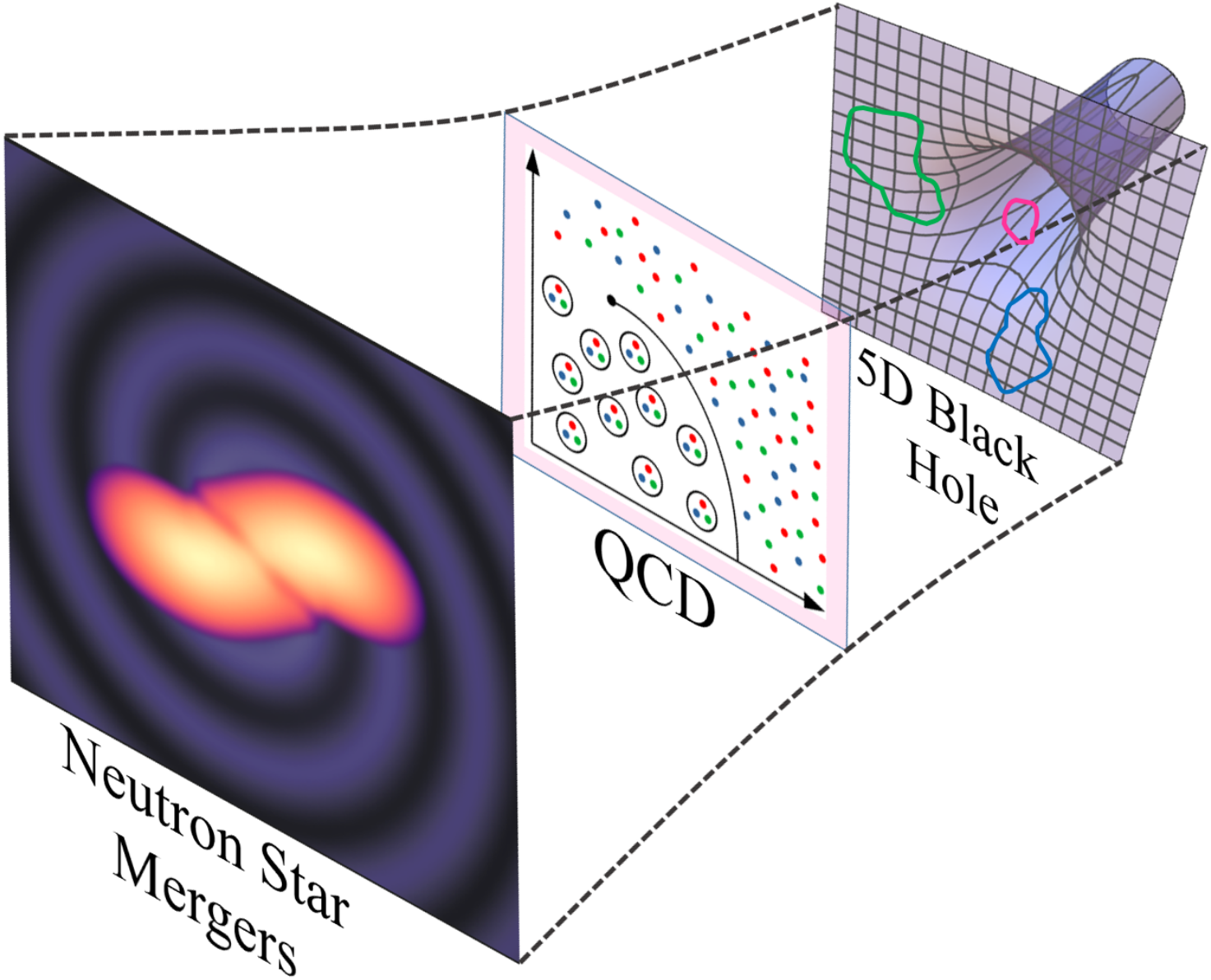


CE, Rezzolla 2209.08101 (MNRAS), see also Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

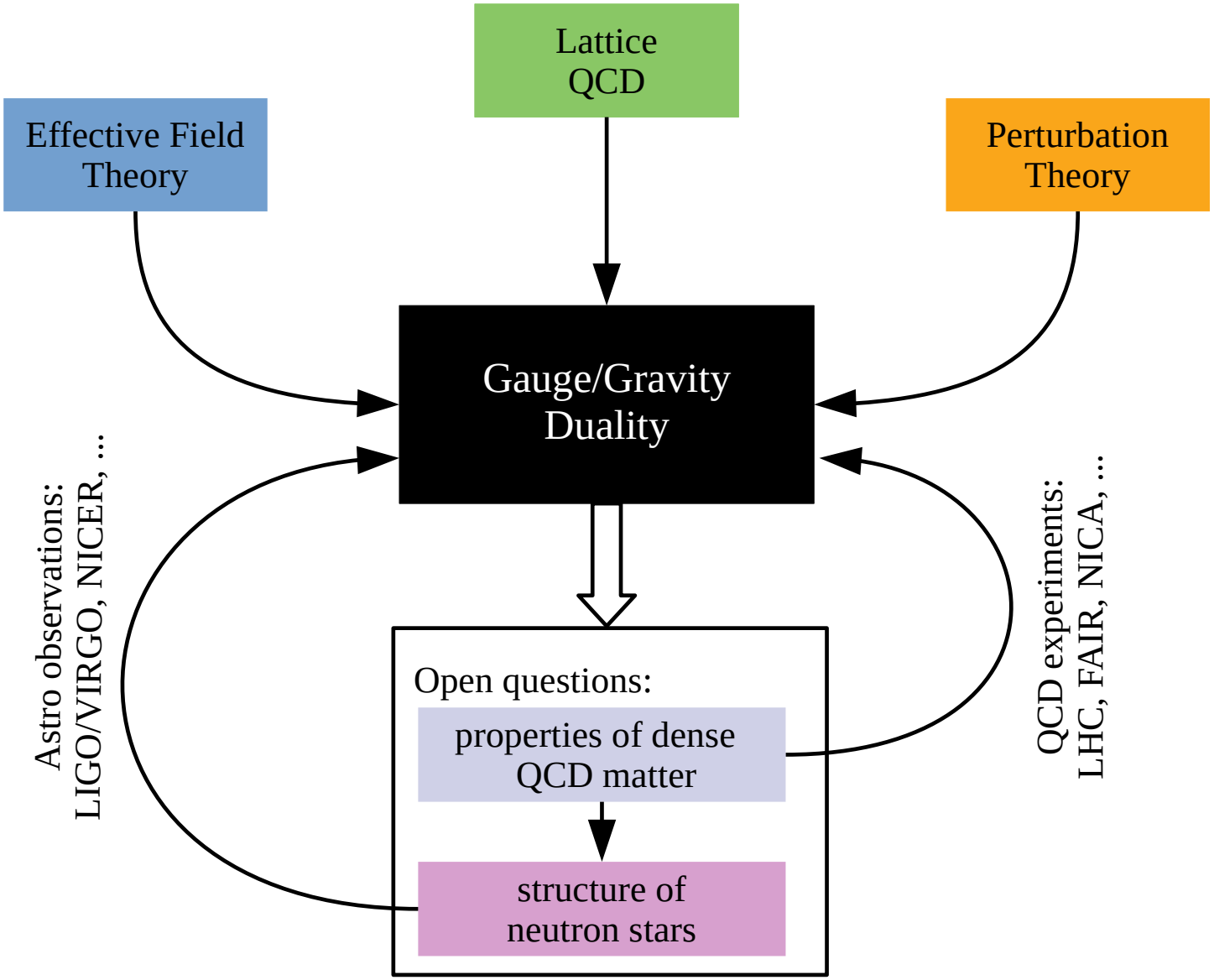
$$\text{GW}: f \approx \text{kHz} \rightarrow f_1, f_2, f_3, \ldots$$
$$\text{EOS}: \; T \gg 0$$
$$\text{lifetime?, composition?, ejecta?}, \ldots$$



A Holographic Approach to BNS Mergers



A Holographic Approach to Dense QCD Matter



Holographic Veneziano QCD

Holographic bottom-up model for QCD with many parameters that are tuned to mimic QCD and that are constrained by lattice QCD data.

Järvinen, Kiritsis 1112.1261 (JHEP); Jokela, Järvinen, Piispa 2405.02394

Gluons: Improved holographic QCD (Einstein-dilaton gravity)

$$S_{\rm g} = N_c^2 M^3 \int d^5x \sqrt{-g} \left[R - \tfrac{4}{3} \tfrac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

$\lambda \equiv e^{\phi} \leftrightarrow \mathrm{Tr} F^2$ sources the 't Hooft coupling in YM theory

Gürsoy, Kiritsis 0707.1324 (JHEP); Gürsoy, Kiritsis, Nitti 0707.1349 (JHEP)

Quarks: Tachyonic Dirac-Born-Infeld (DBI) action

$$S_{\rm f} = -N_f N_c M^3 \int d^5x V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det\left[g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab}\right]}$$

$$F_{rt} = \Phi'(r)\,, \quad \Phi(0) = \mu\,,$$

tachyon $\tau \leftrightarrow \bar{q}q$ controls chiral symmetry breaking.

Bigazzi et al. 0505140 (JHEP); Casero et al. 0702155 (Nucl.Phys.B)

Baryons: homogeneous solution of non-Abelian DBI + Cern–Simons action.

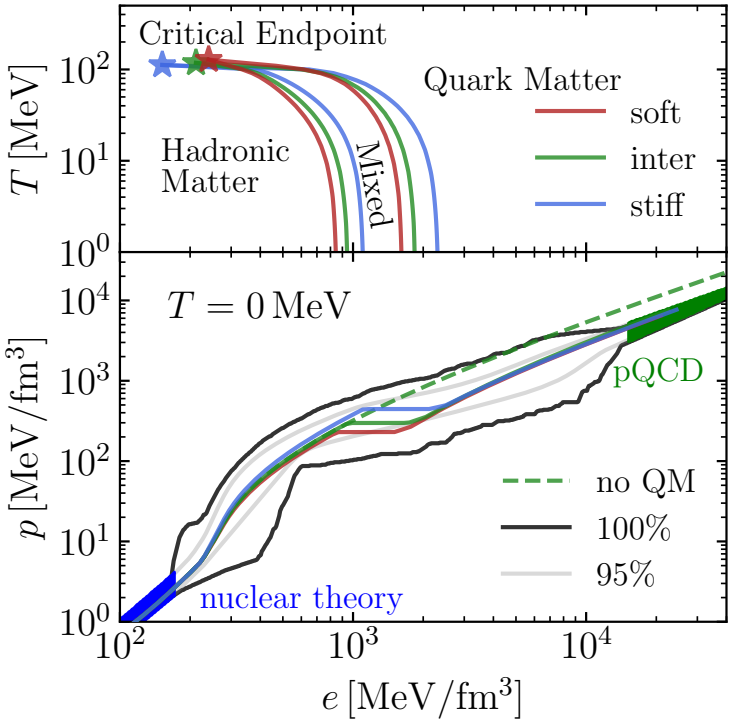
Ishii, Järvinen, Nijs 1903.06169 (JHEP)

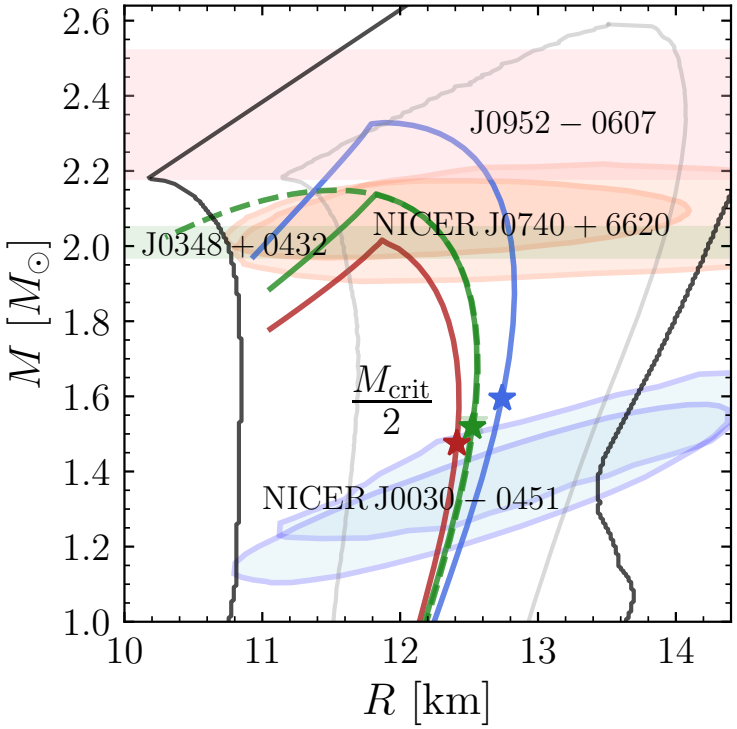
Veneziano limit: $N_c \to \infty$ and $N_f \to \infty$ with $N_f/N_c = \mathcal{O}(1)$ fixed

Järvinen, Kiritsis 1112.1261 (JHEP)

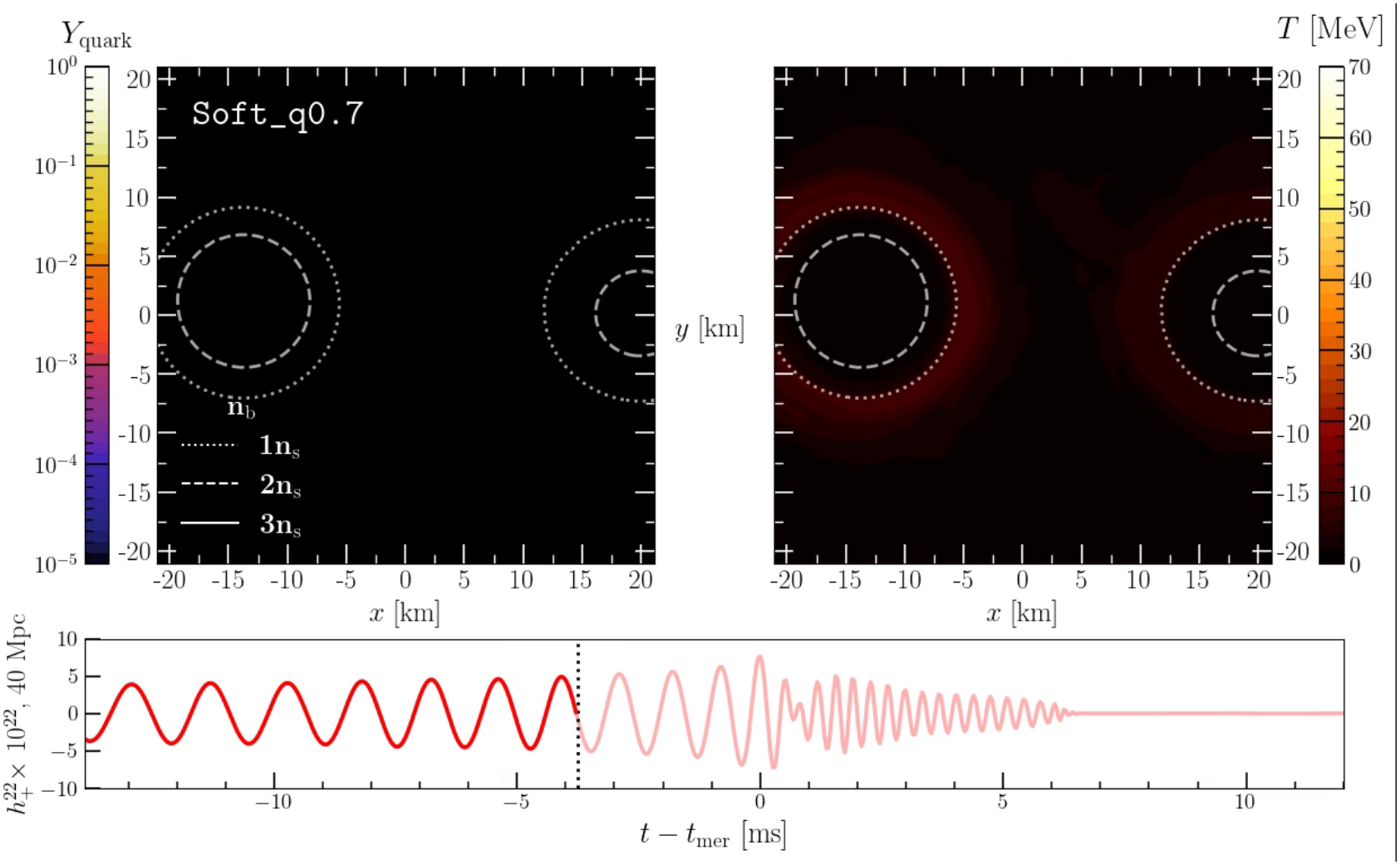
V-QCD Hybrid Equation of State

- Hybrid EOSs: nuclear theory model at lowest densities + V-QCD model for dense baryonic and quark matter at large densities.
- Three tabulated versions on CompOSE database: DEJ(DD2-VQCD) [https://compose.obspm.fr/].



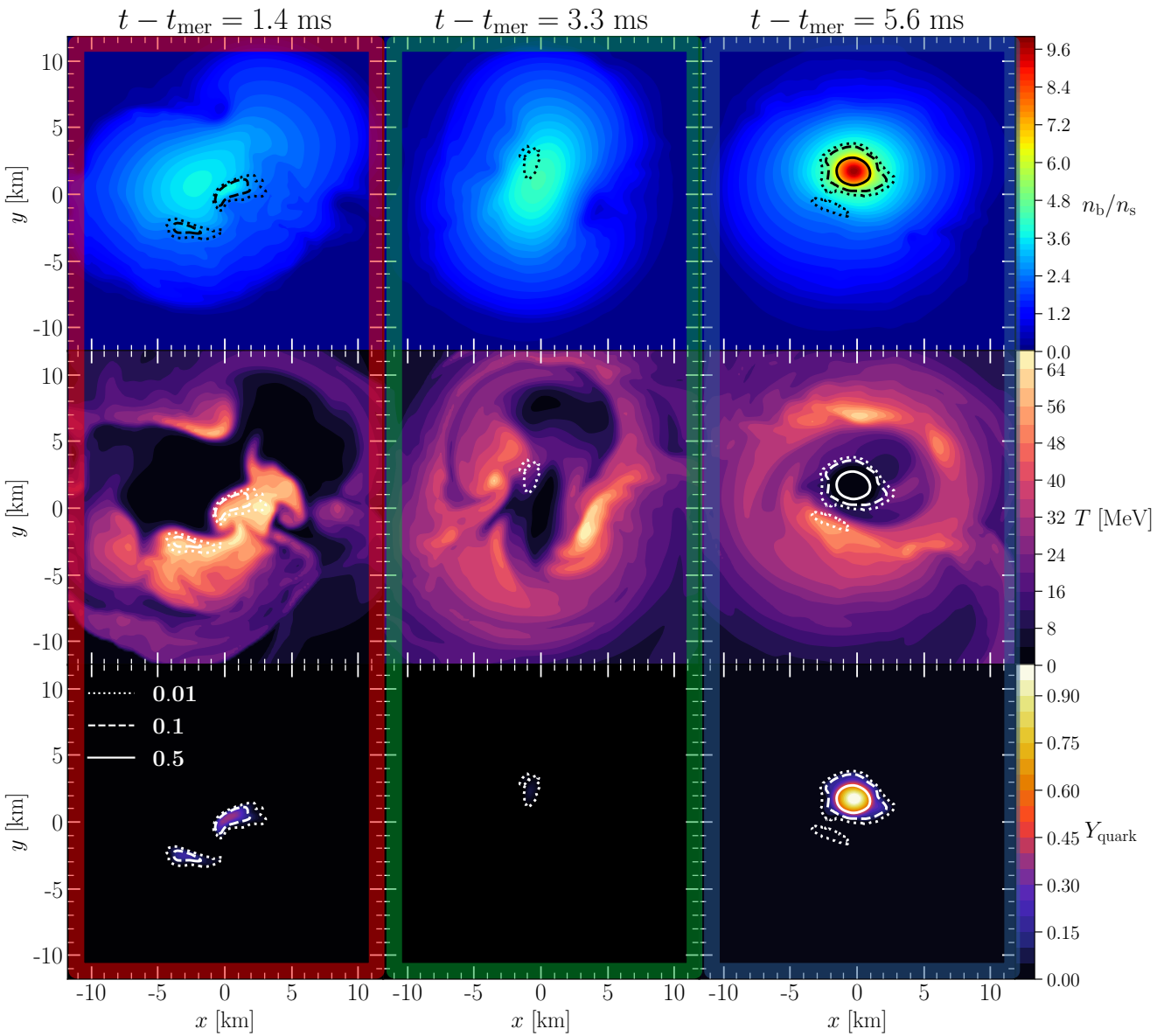


CE, Topolski, Järvinen, Stehr 2402.11013; Demircik, CE, Järvinen 2112.12157 (PRX); Grey band ($M_{\rm TOV} > 2.2\ M_\odot$): CE, Rezzolla 2209.08101 (MNRAS); see also CE, Järvinen, Nijs, van der Schee 1908.03213 (PRD); Jokela, Järvinen, Nijs, Remes 2006.01141 (JHEP)



Initial data with FUKA: Papenfort, Tootle, Grandclement, Most, Rezzolla 2103.09911 (PRD)
Merger simulation with FIL: Most, Papenfort, Rezzolla 1907.10328 (MNRAS)
Implemented in the Einstein Toolkit framework: Löffler et al. 1111.3344 (Class. Quant. Grav.)
HPC project BNSMIC on HPE-Apollo (HAWK) at the HPC Centre in Stuttgart
Movie by Konrad Topolski with data from 2205.05691 (SciPost)

Hot, Warm and Cold Quarks

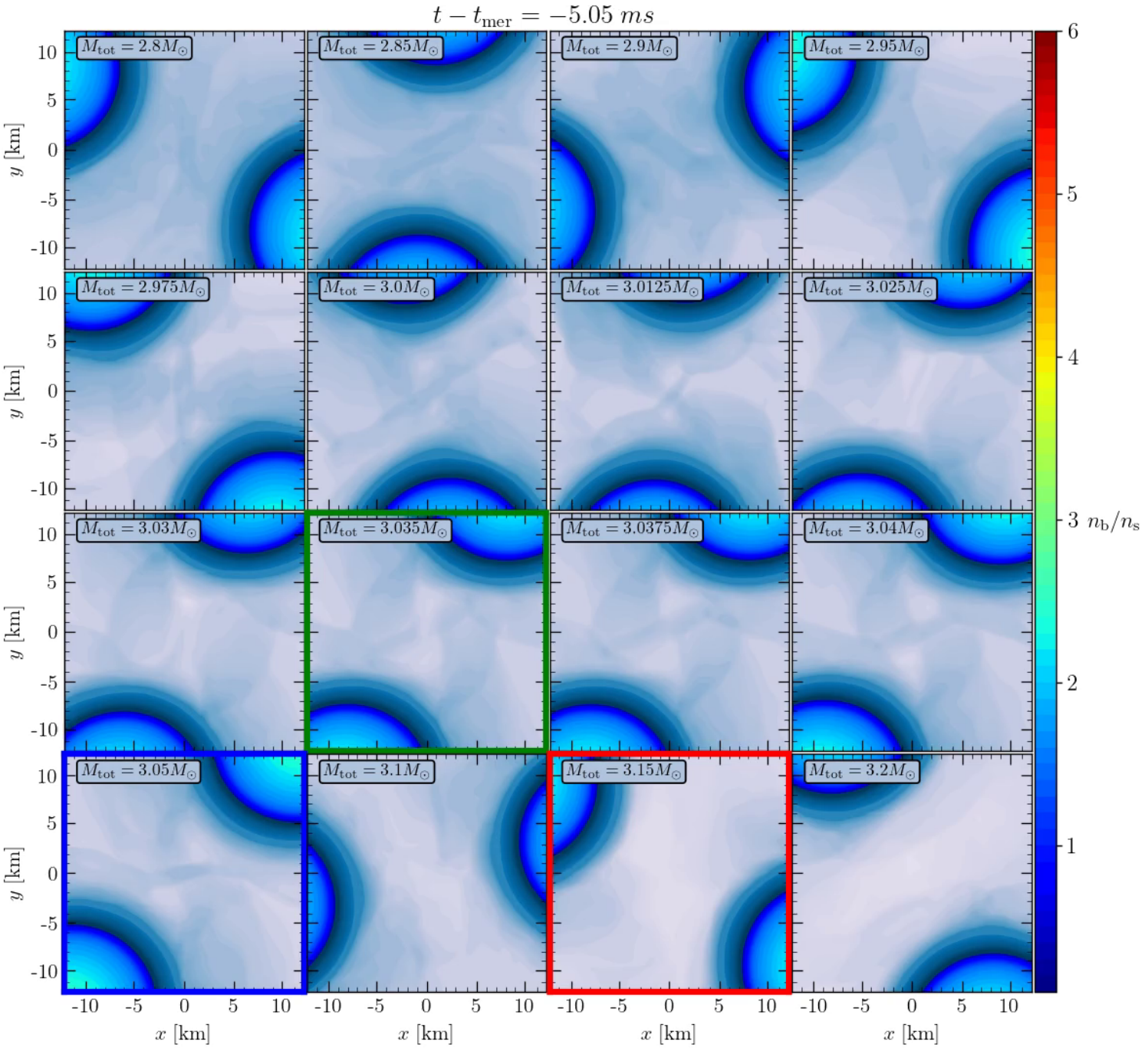


Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla 2205.05691 (SciPost)

Prompt Collapse of a BNS merger

- What do we mean by *"prompt collapse of a BNS merger"*?
- Intuitive answer: black hole is formed *immediately* at the merger.
- Lots of previous work on the *threshold mass* in BNS mergers.

  Bauswein, Baumgarte, Janka 1307.5191 (PRL)
  Köppel, Bovard, Rezzolla 1901.09977 (ApJL)
  Agathos, Zappa, Bernuzzi, Perego, Breschi, Radice 1908.05442 (PRD)
  Bauswein, Blacker, Lioutas, Soultanis, Vijayan, Stergioulas 2010.04461 (PRD)
  Tootle, Papenfort, Most, Rezzolla 2109.00940 (ApJL)
  Kashyap, Das, Radice, Padamata, Prakash, Logoteta, Perego, Godzieba, Bernuzzi, Bombaci, Fattoyev, Reed, Schneider 2111.05183 (PRD),
  Schianchi, Ujevic, Neuweiler, Gieg, Markin, Dietrich 2402.16626
- However, so far no precise criteria for prompt vs. non-prompt collapse.



Movie by Konrad Topolski with data from 2402.11013

Curvature Invariants

- Idea: use curvature invariants for classification of prompt collapse.
- Three principle invariants: Kretschmann, Cern-Pontryagin and Euler scalar

$$K_1 := R_{abcd}\, R^{abcd}\,, \quad K_2 := {}^\star R_{abcd}\, R^{abcd}\,, \quad K_3 := {}^\star R^\star_{abcd}\, R^{abcd}\,. \tag{1}$$

- Useful rewriting in terms of the Weyl tensor

$$C_{abcd} = R_{abcd} - \left(g_{a[c} R_{d]b} - g_{b[c} R_{d]a}\right) + \frac{1}{3} R\; g_{a[c} g_{d]b}\,, \tag{2}$$

$$I_1 = C_{abcd}\, C^{abcd}\,, \quad I_2 = {}^\star C_{abcd}\, C^{abcd}\,, \quad I_3 = {}^\star C^\star{}_{abcd}\, C^{abcd} = -I_1\,. \tag{3}$$

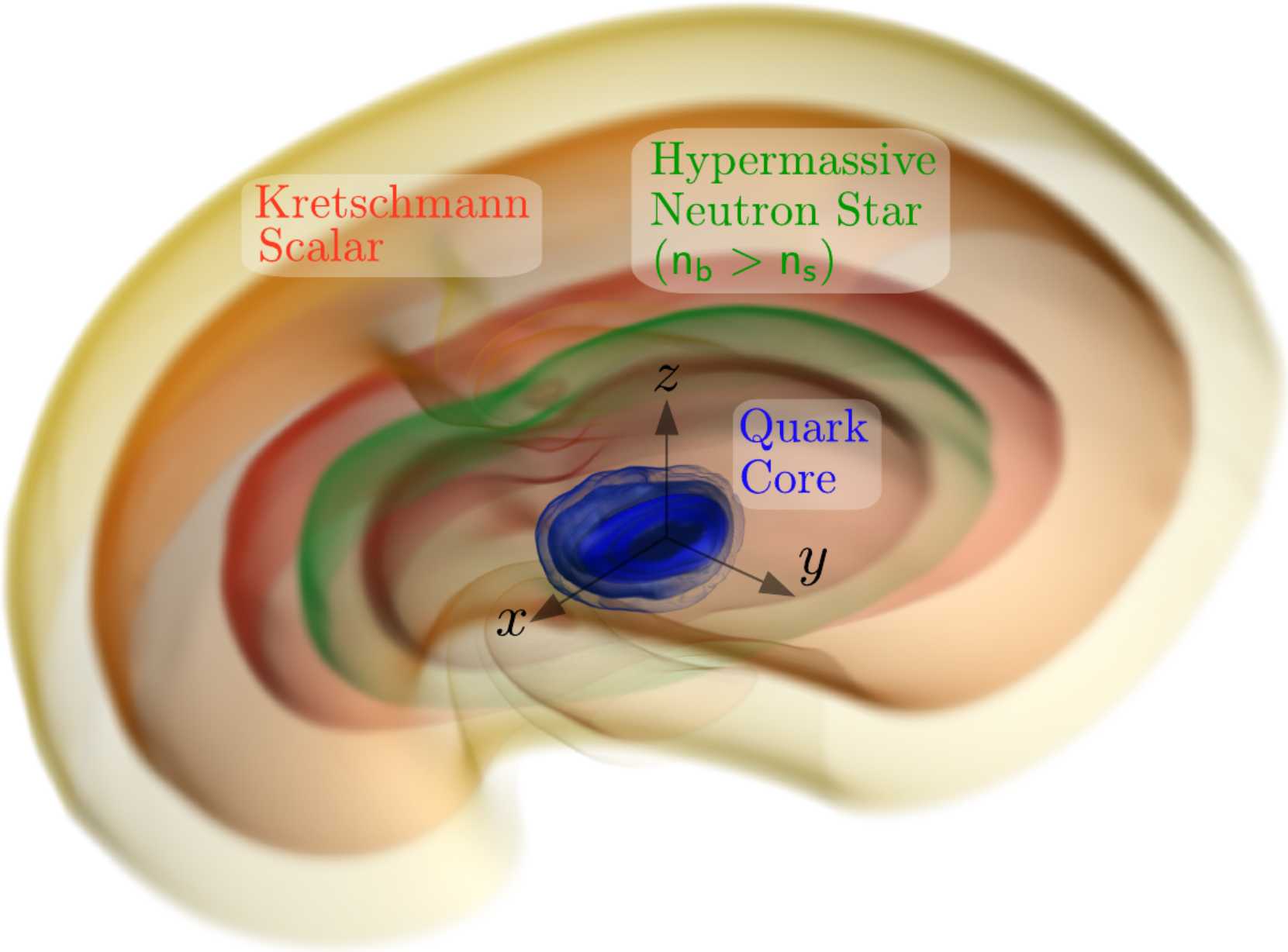
- Simpler in practice to replace Ricci with energy momentum tensor

$$K_1 = I_1 + 128\pi^2 \left(T_{ab} T^{ab} - \frac{1}{6} T^2\right)\,, \quad K_2 = I_2\,, \quad K_3 \approx -K_1\,. \tag{4}$$

- K_1, K_2, K_3 implemented via KRANK library into ETK thorn WeylScalar4.
- Monitor K_1, since $K_3 \approx -K_1$ and negative parity of $K_2(z) = -K_2(-z)$.

Einstein Toolkit (ETK): Löffler et al. 1111.3344 (Class. Quant. Grav.)

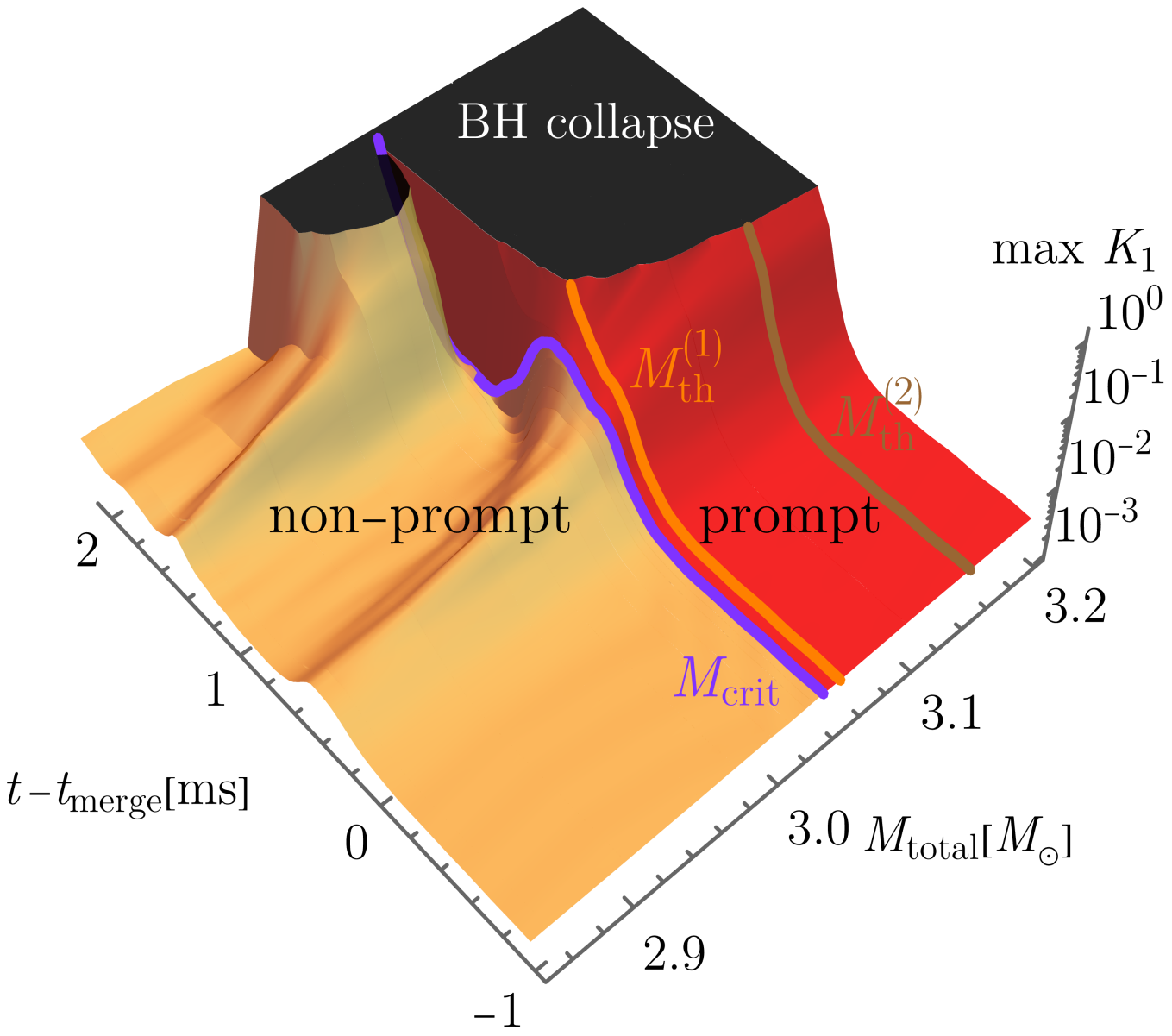
KRANK: Husa, Hinder, Lechner gr-qc/0404023 (Comput. Phys. Commun.)



CE, Topolski, Järvinen, Stehr 2402.11013

Image created with the Thermo Scientific™ Amira 3D visualization software.

Generic Structure of K_1



CE, Topolski, Järvinen, Stehr 2402.11013

Criteria for Prompt Collapse

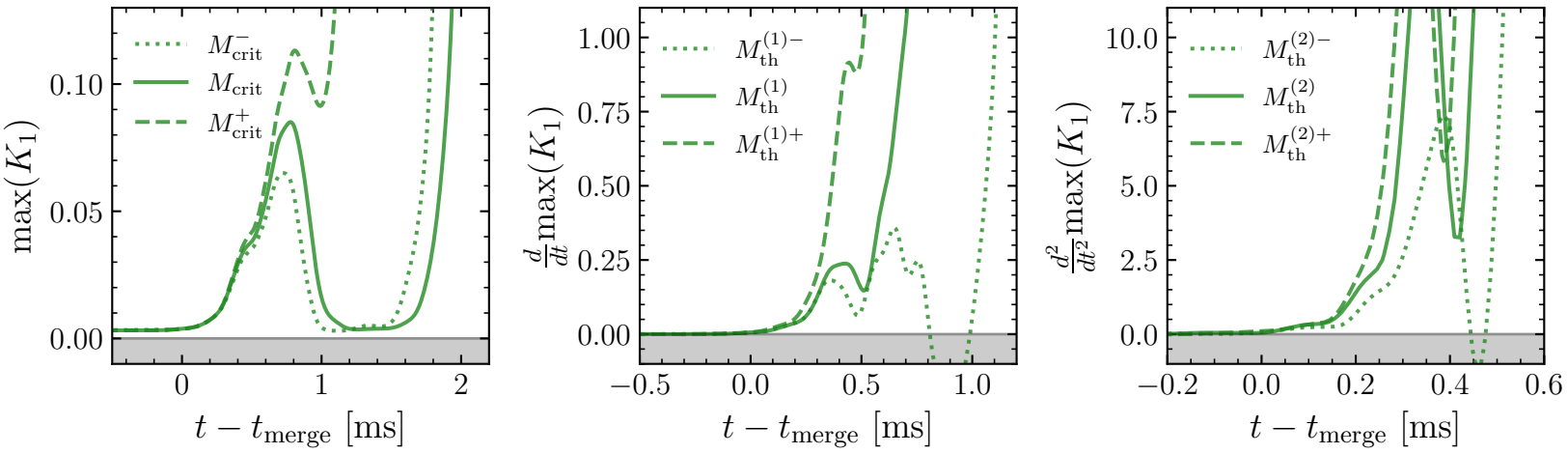
- Generic structure of $\max[K_1(M,t)]$ suggest a clear separation between prompt and non-prompt regime at some *critical mass*

$$M_{\rm crit} = \min(M) : \frac{dt_{\rm crit}}{dM_{\rm total}} < 0 \;\forall\; M_{\rm total} > M\,, \tag{5}$$

where $t_{\rm crit}(M_{\rm total})$ is the time at which an apparent horizon is formed.

- *Threshold masses of promptness p* characterize how strongly gravitational pull dominates over matter repulsion and overall collision dynamics

$$\left\{ M_{\rm th}^{(p)} = \min(M_{\rm total}) : \frac{d^p}{dt^p}\max(K_1) \geqslant 0 \;\forall\; t > t_{\rm merge} \right\}\,. \tag{6}$$



CE, Topolski, Järvinen, Stehr 2402.11013

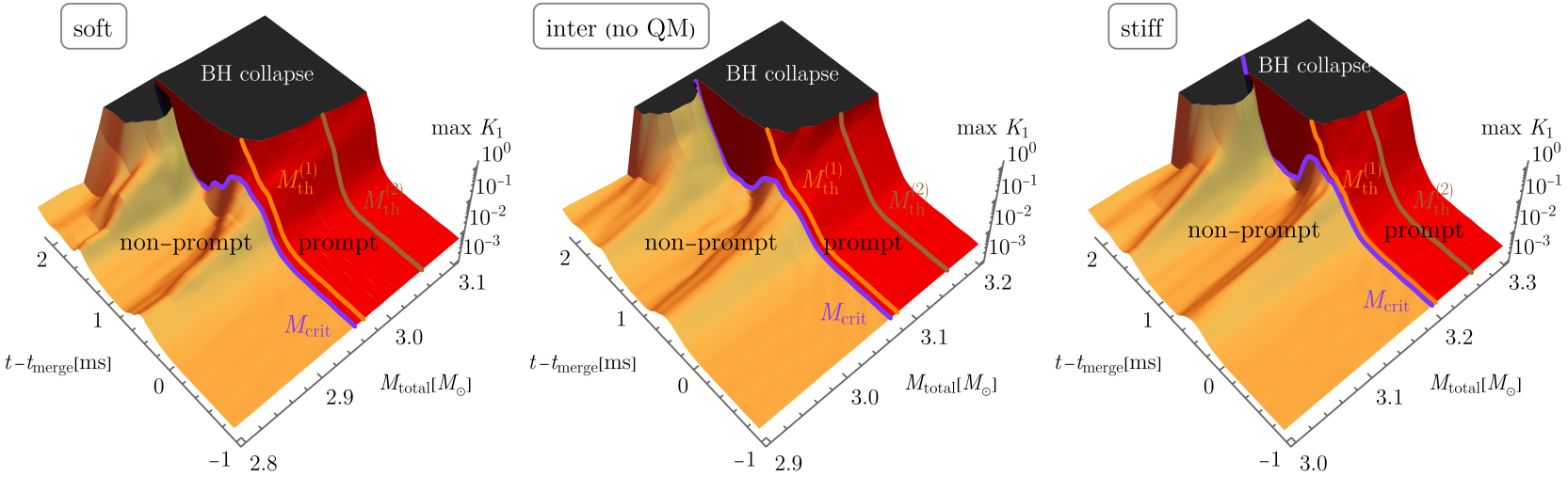
Scaling with the EOS Stiffness

- Soft to stiff EOS variants, i.e., small to large M_{TOV} and maximum c_s^2

$$2.95 M_\odot \lesssim M_{\mathrm{crit}} \lesssim 3.19 M_\odot \,. \tag{7}$$

- Consistent with non-prompt collapse outcome of GW170817 with $M_{\mathrm{total}}^{\mathrm{GW170817}} \approx 2.74\ M_\odot < M_{\mathrm{crit}}$, as suggested by observed EM emission.
- Rather tight quasi-universal mass-ratio:

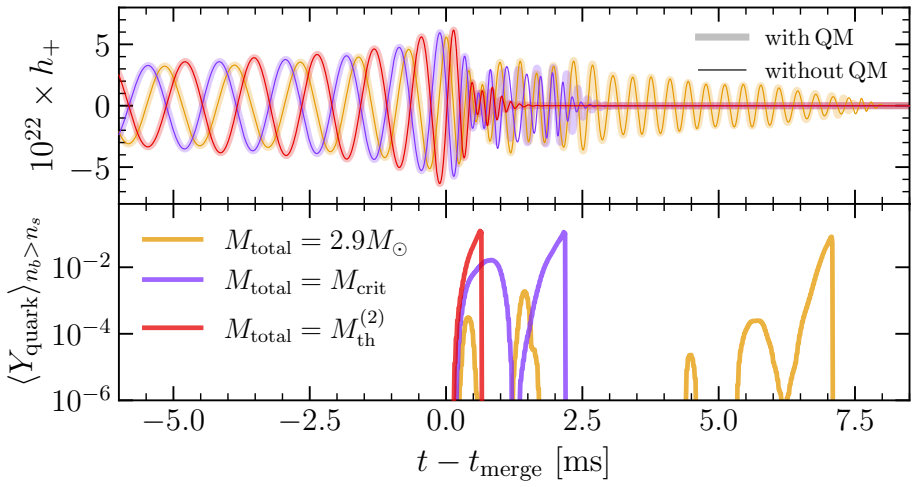
$$M_{\mathrm{crit}}/M_{\mathrm{TOV}} \approx 1.41 \pm 0.06 \tag{8}$$

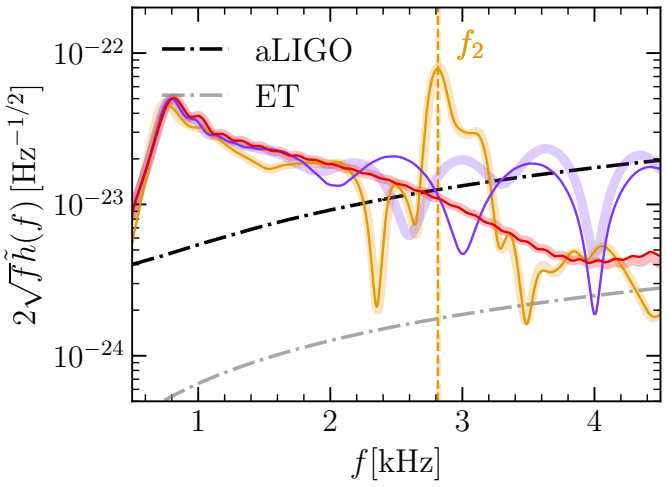


CE, Topolski, Järvinen, Stehr 2402.11013

Gravitational Waves

- Periodic recurrence of quark matter inside sub-critical HMNSs.
- Formation of sizeable quark-matter core induces collapse.
- Significant imprint of quark matter on GW spectrum only close to M_{crit}.
- Maybe EM counterpart is a more promising probe for quark matter?

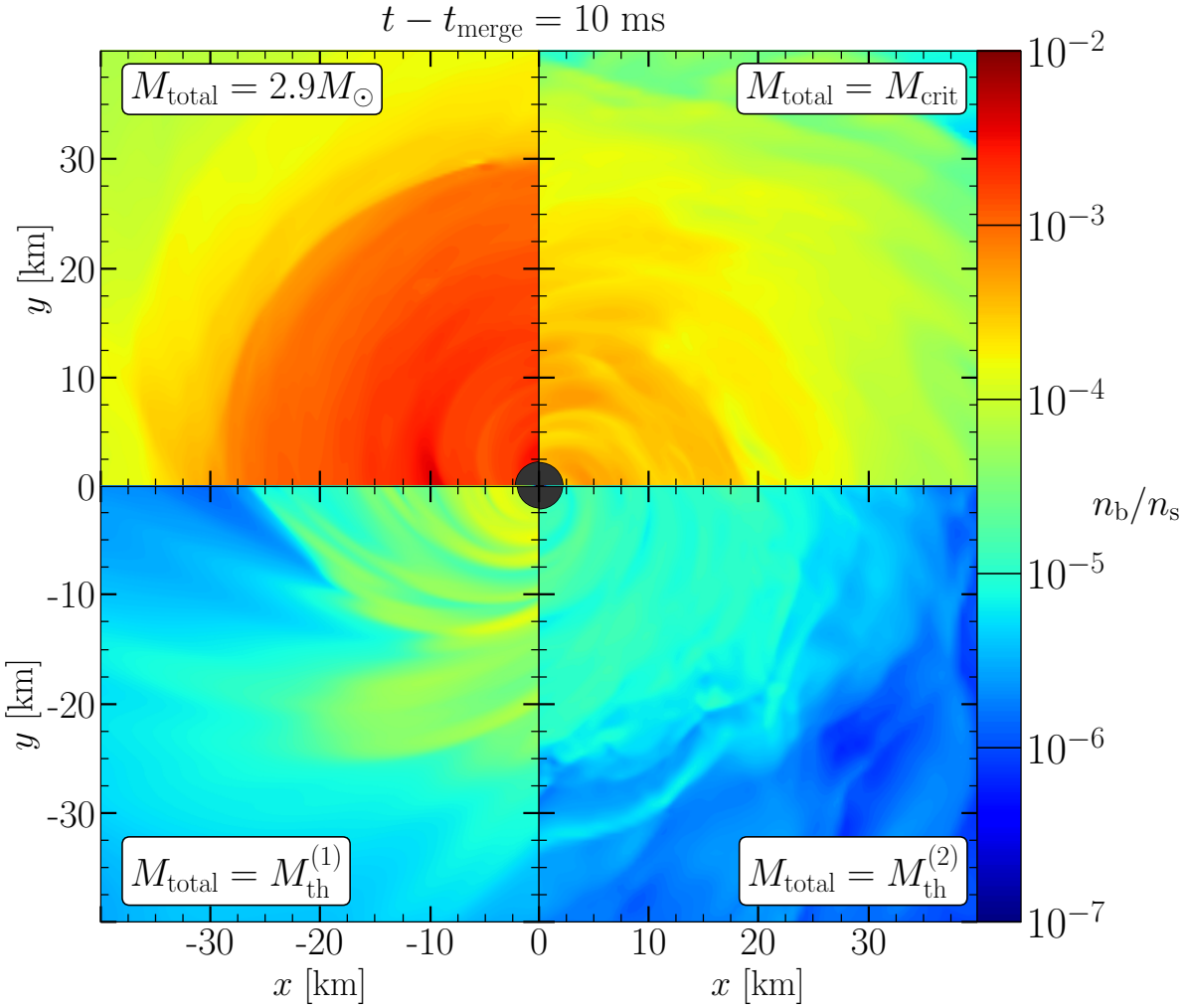




CE, Topolski, Järvinen, Stehr 2402.11013

Residual Matter

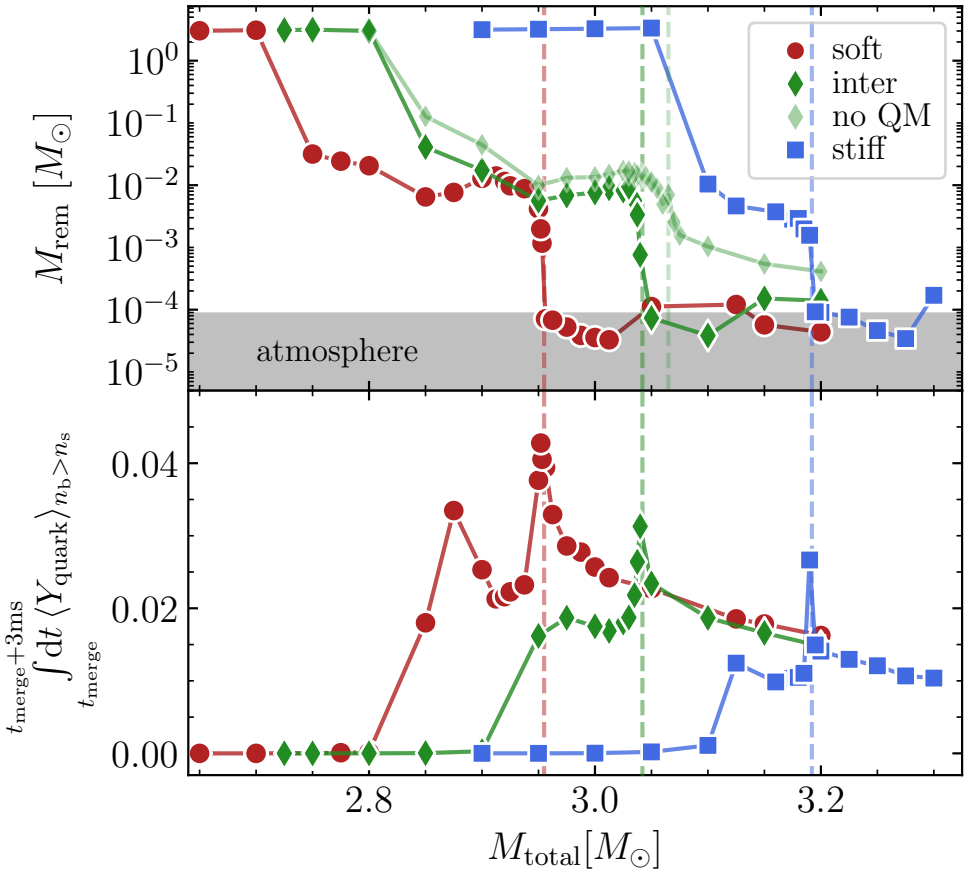
- EM counterpart needs matter to "shine".
- Significant drop in residual matter outside horizon at M_{crit}.



CE, Topolski, Järvinen, Stehr 2402.11013

Impact of Quark Matter

- **With quark matter:** essentially all matter is swallowed by the black hole in super-critical mergers ($M_{\rm total} > M_{\rm crit}$).
- Most quark matter formed in critical-mass mergers ($M_{\rm total} = M_{\rm crit}$).
- **Without quark matter:** some residual matter even for $M_{\rm total} > M_{\rm crit}$.
- Soft quark-matter cores collapse quicker and clear exterior more efficiently.



Summary

- Basic properties ($p(e)\,, M(R)\,, c_s^2\,, \tilde{\Lambda}(\mathcal{M}_{\rm chirp})\,, \ldots$) of the cold EOS and neutron stars from constraint sampling.

  CE, Rezzolla arXiv:2209.08101 (MNRAS), arXiv:2207.04417 (ApJL);
  Altiparmak, CE, Rezzolla arXiv:2203.14974 (ApJL)

- Hot V-QCD hybrid EOS with deconfinement phase transition, three tabulated versions on CompOSE database: DEJ(DD2-VQCD).

  Demircik, CE, Järvinen 2112.12157 (PRX), https://compose.obspm.fr/

- Three different stages in HMNSs: hot, warm, cold quark matter.

  Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla 2205.05691 (SciPost)

- Attempt to classify prompt collapse with curvature invariants: $M_{\rm crit}, M_{\rm th}^{(p)}$.

  CE, Topolski, Järvinen, Stehr 2402.11013

- Relation between critical mass and TOV-mass: $M_{\rm crit}/M_{\rm TOV} \approx 1.41 \pm 0.06$
- Most quark matter formed in critical-mass mergers: $M_{\rm total} = M_{\rm crit}$.
- Small impact on GW spectrum, but maybe visible from EM counterpart.

Threshold Mass from Newtonian Free-Fall

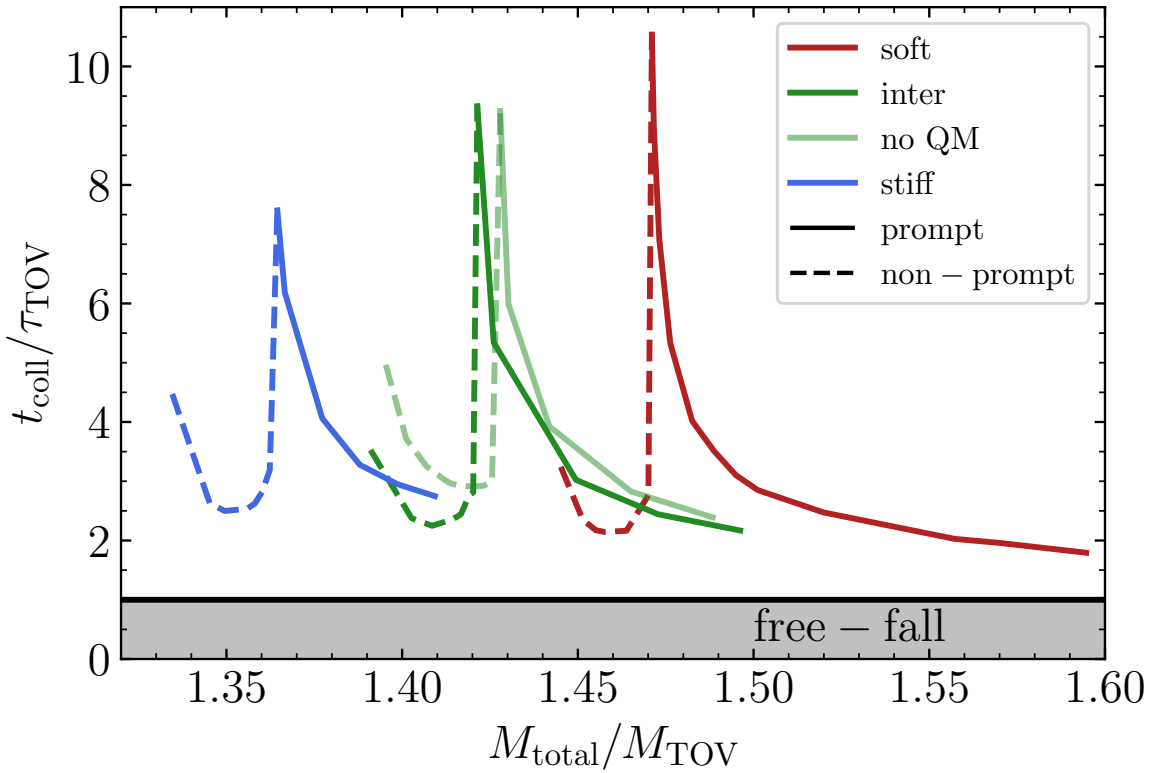
- Definition in terms of minimum lapse $\alpha(t)$ and Newtonian free-fall time

$$M/M_{\rm TOV} \to M_{\rm th}/M_{\rm TOV} \text{ for } t_{\rm coll} \to \tau_{\rm TOV}\,, \quad \tau_{\rm TOV} = \frac{\pi}{2}\sqrt{\frac{R^3_{\rm TOV}}{2\,M_{\rm TOV}}}\,. \quad (9)$$

$$\bar{t}_{\rm merge}:\ \min(\alpha(t)) = 0.35\,, \quad t_{\rm BH}:\ \min(\alpha(t)) = 0.2\,. \quad (10)$$

Köppel, Bovard, Rezzolla 1901.09977 (ApJL)

- However, finite pressure fluids never reach Newtonian free-fall limit.



CE, Topolski, Järvinen, Stehr 2402.11013