Probing the Phase Diagram and Equation of State of QCD in Neutron Star Mergers: From Prompt Collapse to Long Ringdown Signals

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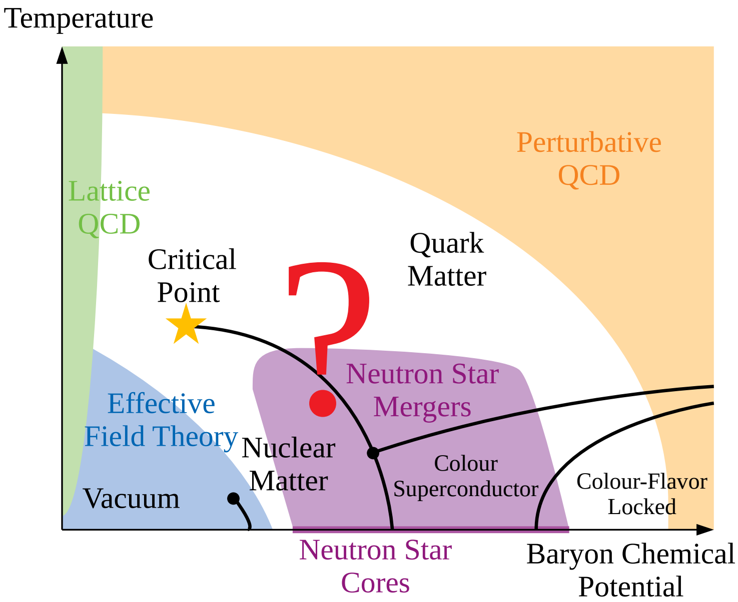
AEI Potsdam, 9 October 2024

Based on 2402.11013 with
Matti Järvinen, Konrad Topolski and Alina Stehr

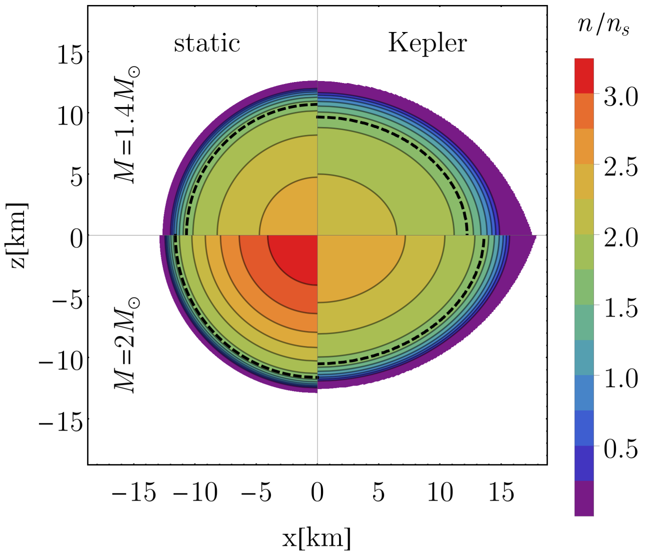
and 2403.03246 with
Tyler Gorda, Aleksi Kurkela and Luciano Rezzolla

Introduction

What we (don't) know about QCD



Density Profile



Demircik, CE, Järvinen 2009.10731 (ApJL)

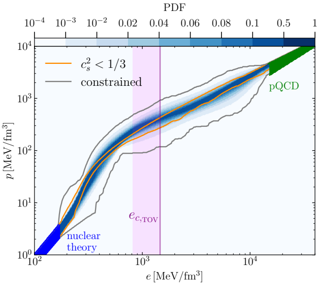
Equation of State

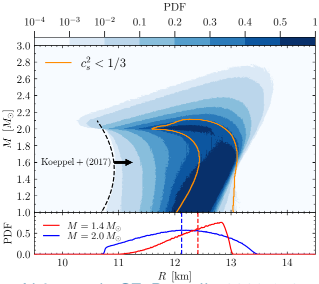
Relates properties of dense matter to properties of neutron stars.

$$p = p(e, \ldots) \xrightarrow{\text{Einstein Eq.}} M(R)$$

Constrained by theory and neutron star observations:

- Causality & thermodynamic stability: $0 < c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} < 1$
- Direct mass and radius measurements: $M_{\mathrm{TOV}} \gtrsim 2\,M_\odot$, $11\,\mathrm{km} \lesssim R \lesssim 14\,\mathrm{km}$
- First GW detection from a BNS merger GW170817: $\tilde{\Lambda} \lesssim 720$



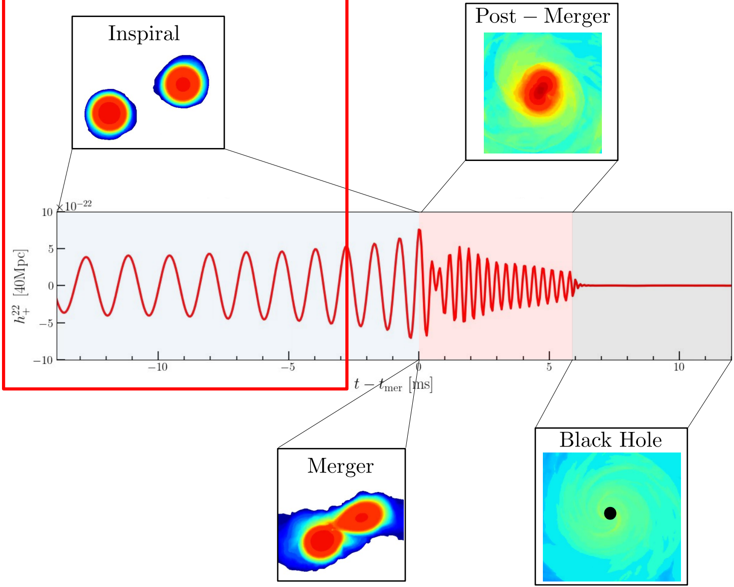


Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

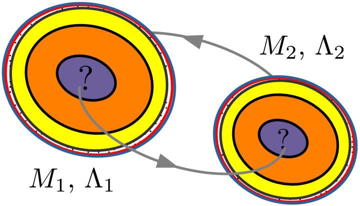
Lots of recent work: . . . ; Annala, Gorda, Kurkela, Nättilä, Vuorinen 2105.05132 (PRX); Komoltsev, Kurkela 2111.05350 (PRL); Altiparmak, CE, Rezzolla (ApJL) 2203.14974; Gorda, Komoltsev, Kurkela 2204.11877; CE, Rezzolla 2207.04417 (ApJL); . . .

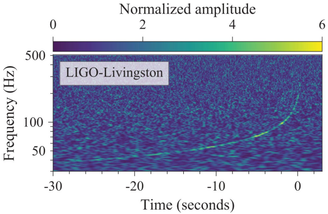
$\mathrm{GW} : f \approx 100s\ \mathrm{Hz} \rightarrow \mathcal{M}_{\mathrm{chirp}}$

$\mathrm{EOS} : \ T \approx 0 \rightarrow \tilde{\Lambda}$



Chirp Mass and Binary Tidal Deformability





- Chirp mass can be extracted accurately from inspiral GW strain frequency

$$\frac{\mathrm{d}f_{GW}}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\mathcal{M}_{chirp}^{5/3} f_{GW}^{11/3}\,, \quad \mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}\,.$$

Cutler, Flanagan 9402014 (PRD)

- Binary tidal deformability encodes EOS properties via Λ_i

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12M_2+M_1)\,M_1^4\Lambda_1 + (12M_1+M_2)\,M_2^4\Lambda_2}{(M_1+M_2)^5}\,, \quad \Lambda_i = \tfrac{2}{3}k_2\,(R_i/M_i)^5\,.$$

- GW170817 "best" event so far:

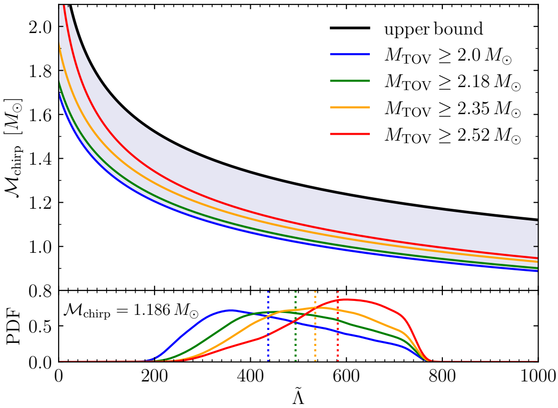
$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002} M_\odot, \quad q = M_2/M_1 > 0.7, \quad \tilde{\Lambda} \lesssim 720\,.$$

LIGO/Virgo 1805.11579 (PRX)

Binary Tidal Deformability

More than 10^8 binary configurations with $M_{1,2} \in [0.5 M_\odot, M_{\rm TOV}]$.

$$\tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}^{c}_{chirp} \longrightarrow \tilde{\Lambda}^{\rm min}_{1.186} \in [236, 301]\,,\ M_{\rm TOV} > [2.18, 2.52]\ M_\odot\,.$$

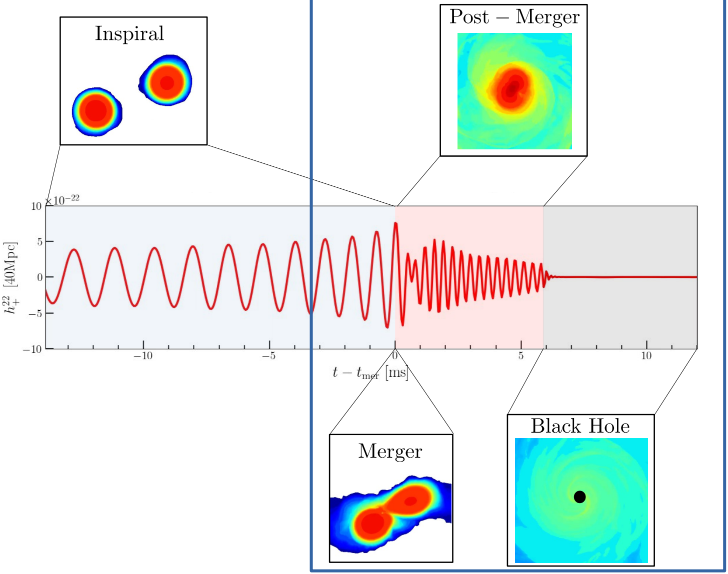


CE, Rezzolla 2209.08101 (MNRAS), see also Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

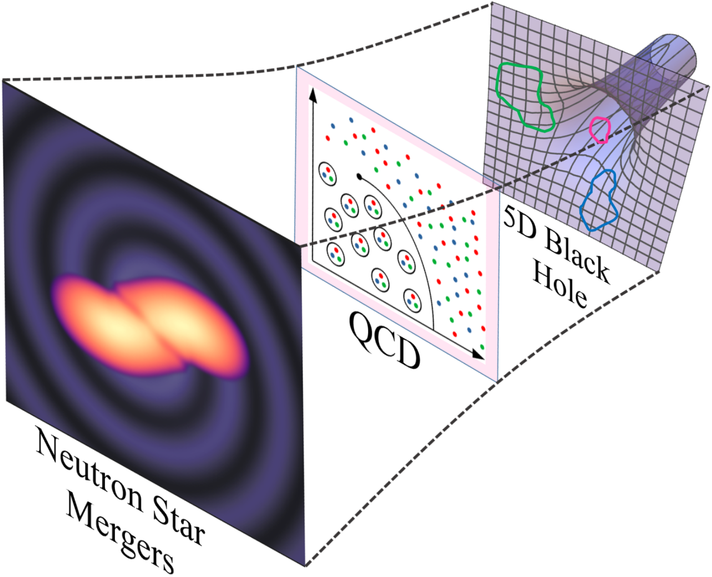
GW : $f \approx \mathrm{kHz} \rightarrow f_1, f_2, f_3, \ldots$

EOS : $T \gg 0$

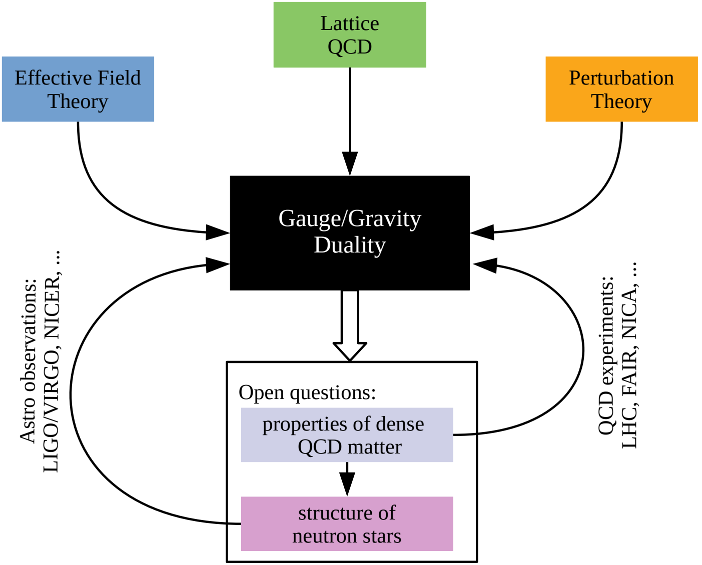
lifetime?, composition?, ejecta?, . . .



A Holographic Approach to BNS Mergers



A Holographic Approach to Dense QCD Matter



Holographic Veneziano QCD

Holographic bottom-up model for QCD with many parameters that are tuned to mimic QCD and that are constrained by lattice QCD data.

Järvinen, Kiritsis 1112.1261 (JHEP); Jokela, Järvinen, Piispa 2405.02394

Gluons: Improved holographic QCD (Einstein-dilaton gravity)

$$S_{\rm g} = N_c^2 M^3 \int d^5x\sqrt{-g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right]$$

$\lambda \equiv e^{\phi} \leftrightarrow \mathrm{Tr}F^2$ sources the 't Hooft coupling in YM theory

Gürsoy, Kiritsis 0707.1324 (JHEP); Gürsoy, Kiritsis, Nitti 0707.1349 (JHEP)

Quarks: Tachyonic Dirac-Born-Infeld (DBI) action

$$S_{\rm f} = -N_f N_c M^3 \int d^5x V_{f0}(\lambda)e^{-\tau^2}\sqrt{-\det\left[g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab}\right]}$$

$$F_{rt} = \Phi'(r)\,, \quad \Phi(0) = \mu\,,$$

tachyon $\tau \leftrightarrow \bar{q}q$ controls chiral symmetry breaking.

Bigazzi et al. 0505140 (JHEP); Casero et al. 0702155 (Nucl.Phys.B)

Baryons: homogeneous solution of non-Abelian DBI + Cern–Simons action.

Ishii, Järvinen, Nijs 1903.06169 (JHEP)

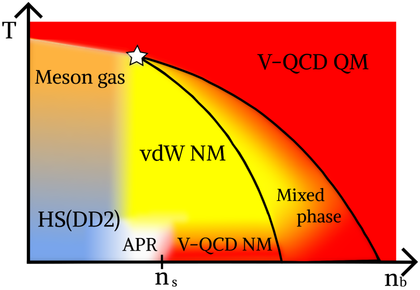
Veneziano limit: $N_c \to \infty$ and $N_f \to \infty$ with $N_f/N_c = \mathcal{O}(1)$ fixed

Järvinen, Kiritsis 1112.1261 (JHEP)

V-QCD Hybrid Equation of State

- Computing neutron star properties requires EOS at low and high densities.
- Homogeneous approx. for V-QCD baryons not reliable at low densities.
- Hybrid EOSs: nuclear theory model at lowest densities + V-QCD model for dense baryonic and quark matter at large densities.
- Van der Waals construction to extend cold V-QCD baryons to finite-T.
  Vovchenko, Motornenko, Alba, Gorenstein, Satarov, Stoecker 1707.09215 (PRC)
- Mixed baryon and quark matter phase from mech. and chem. equilibrium.

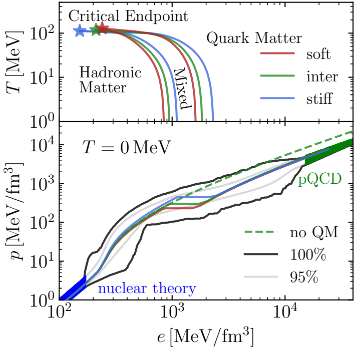
$$X_{NM}(T, n_b^{(1)}, Y_q^{(1)}) = X_{QM}(T, n_b^{(2)}, Y_q^{(2)}), \quad X = \{p, \mu_b, \mu_{le}\}.$$

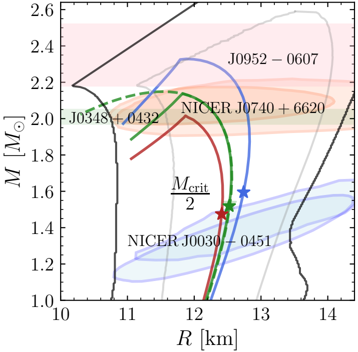


Demircik, CE, Järvinen 2112.12157 (PRX);

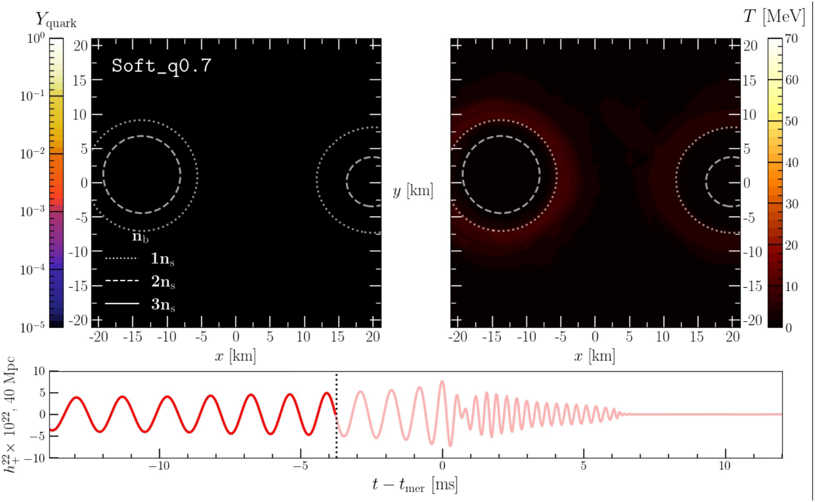
Equation of State and M-R relation

- By construction good agreement with credibility intervals of generic EOS parametrizations and astrophysical constraints.
- Three tabulated versions on CompOSE database: DEJ(DD2-VQCD) [https://compose.obspm.fr/].



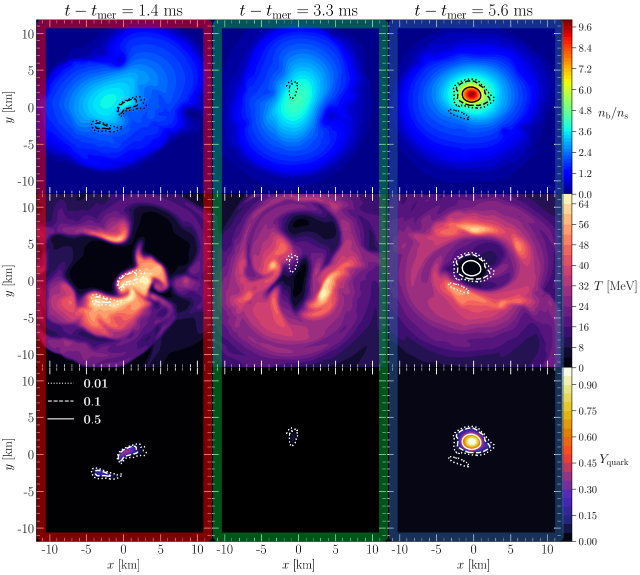


CE, Topolski, Järvinen, Stehr 2402.11013; Demircik, CE, Järvinen 2112.12157 (PRX); Grey band ($M_{\rm TOV} > 2.2\ M_\odot$): CE, Rezzolla 2209.08101 (MNRAS); see also CE, Järvinen, Nijs, van der Schee 1908.03213 (PRD); Jokela, Järvinen, Nijs, Remes 2006.01141 (JHEP)



Initial data with FUKA: Papenfort, Tootle, Grandclement, Most, Rezzolla 2103.09911 (PRD)
Merger simulation with FIL: Most, Papenfort, Rezzolla 1907.10328 (MNRAS)
Implemented in the Einstein Toolkit framework: Löffler et al. 1111.3344 (Class. Quant. Grav.)
HPC project BNSMIC on HPE-Apollo (HAWK) at the HPC Centre in Stuttgart
Movie by Konrad Topolski with data from 2205.05691 (SciPost)

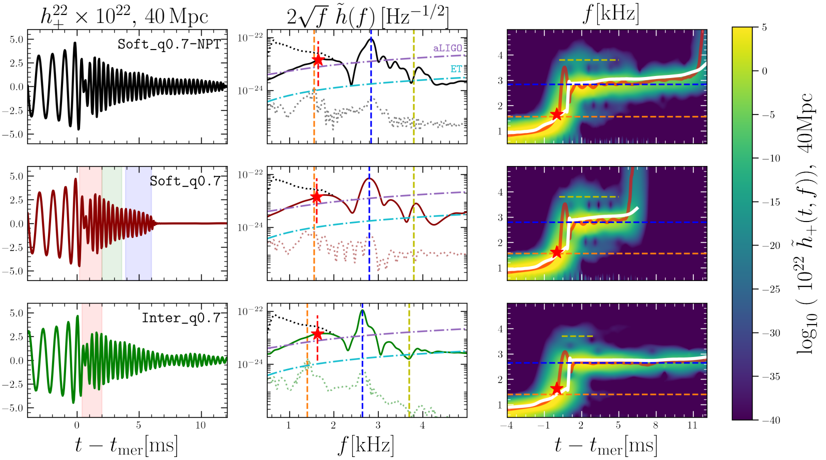
Hot, Warm and Cold Quarks



Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla 2205.05691 (SciPost)

Waveforms and Frequency Spectra

- Small impact of quark matter on post-merger frequencies.
- Phase transition triggered collapse (PTTC) leads to shorter lifetime of the hyper massive neutron star (HMNS).



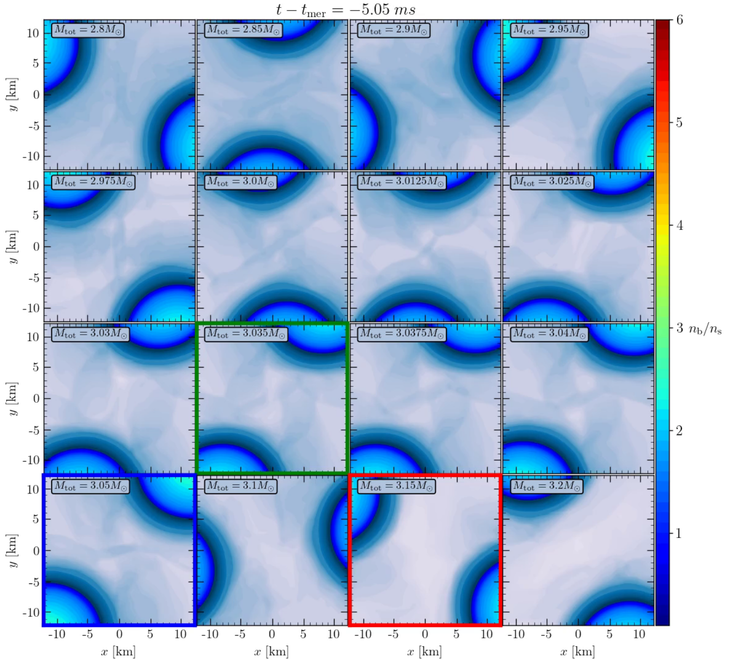
Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla 2205.05691 (SciPost)

Prompt Black Hole Formation in Binary Neutron Star Mergers

Prompt Collapse of a BNS merger

- What do we mean by *"prompt collapse of a BNS merger"*?
- Intuitive answer: black hole is formed *immediately* at the merger.
- Vast literature using different criteria to estimate the *threshold mass*.

  Bauswein, Baumgarte, Janka 1307.5191 (PRL)
  Köppel, Bovard, Rezzolla 1901.09977 (ApJL)
  Kiuchi, Kyutoku, Shibata, Taniguchi 1903.01466 (ApJL)
  Agathos, Zappa, Bernuzzi, Perego, Breschi, Radice 1908.05442 (PRD)
  Bauswein, Blacker, Lioutas, Soultanis, Vijayan, Stergioulas 2010.04461 (PRD)
  Tootle, Papenfort, Most, Rezzolla 2109.00940 (ApJL)
  Kashyap, Das, Radice, Padamata, Prakash, Logoteta, Perego, Godzieba, Bernuzzi, Bombaci, Fattoyev, Reed, Schneider 2111.05183 (PRD)
  Perego, Logoteta, Radice, Bernuzzi, Kashyap, Das, Padamata, Prakash 2112.05864 (PRL)
  Kölsch, Dietrich, Ujevic, Bruegmann 2112.11851 (PRD)
  Schianchi, Ujevic, Neuweiler, Gieg, Markin, Dietrich 2402.16626



Movie by Konrad Topolski with data from 2402.11013

Curvature Invariants

- Idea: use curvature invariants for classification of prompt collapse.
- Three principle invariants: Kretschmann, Cern-Pontryagin and Euler scalar

$$K_1 := R_{abcd}\, R^{abcd} \,, \quad K_2 := {}^\star R_{abcd}\, R^{abcd} \,, \quad K_3 := {}^\star R^\star_{abcd}\, R^{abcd} \,. \qquad (1)$$

- Useful rewriting in terms of the Weyl tensor

$$C_{abcd} = R_{abcd} - \left(g_{a[c} R_{d]b} - g_{b[c} R_{d]a}\right) + \frac{1}{3} R \; g_{a[c} g_{d]b} \,, \qquad (2)$$

$$I_1 = C_{abcd}\, C^{abcd} \,, \quad I_2 = {}^\star C_{abcd}\, C^{abcd} \,, \quad I_3 = {}^\star C^\star{}_{abcd}\, C^{abcd} = -I_1 \,. \qquad (3)$$

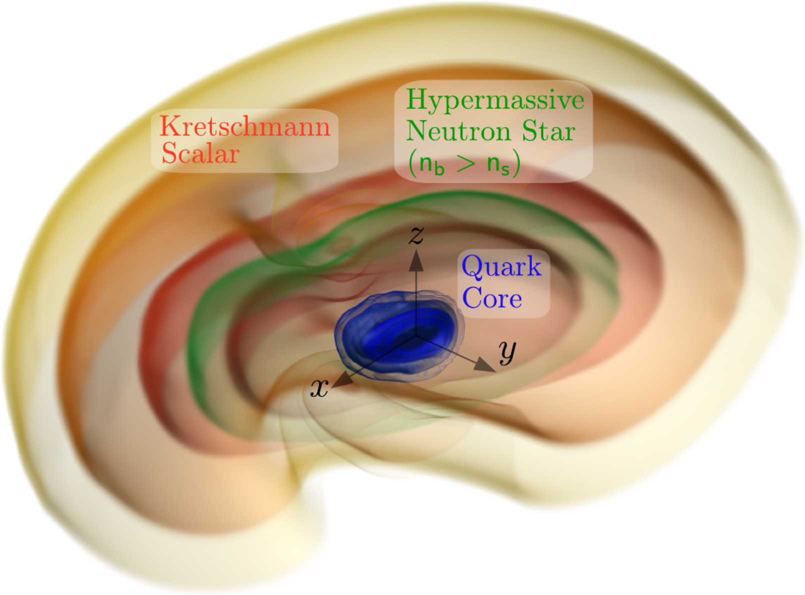
- Simpler in practice to replace Ricci with energy momentum tensor

$$K_1 = I_1 + 128\pi^2 \left(T_{ab} T^{ab} - \frac{1}{6} T^2 \right) \,, \quad K_2 = I_2 \,, \quad K_3 \approx -K_1 \,. \qquad (4)$$

- K_1, K_2, K_3 implemented via KRANK library into ETK thorn WeylScalar4.
- Monitor K_1, since $K_3 \approx -K_1$ and negative parity of $K_2(z) = -K_2(-z)$.

Einstein Toolkit (ETK): Löffler et al. 1111.3344 (Class. Quant. Grav.)

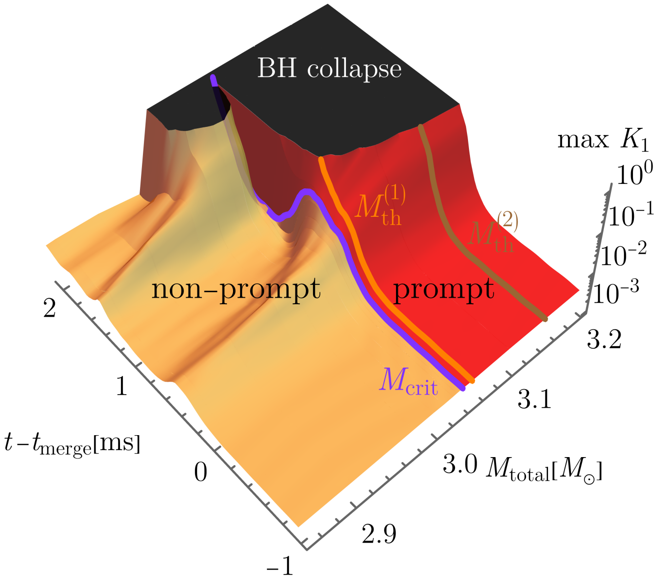
KRANK: Husa, Hinder, Lechner gr-qc/0404023 (Comput. Phys. Commun.)



CE, Topolski, Järvinen, Stehr 2402.11013

Image created with the Thermo Scientific™ Amira 3D visualization software.

Generic Structure of K_1



CE, Topolski, Järvinen, Stehr 2402.11013

Criteria for Prompt Collapse

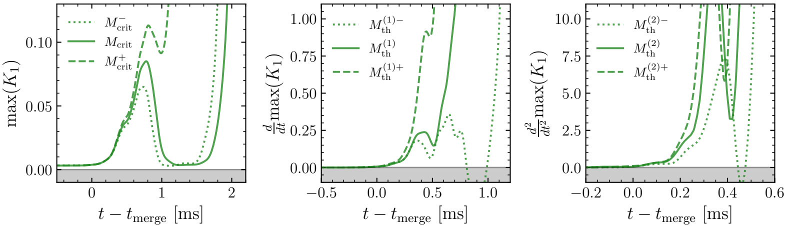
- Generic structure of $\max[K_1(M,t)]$ suggest a clear separation between prompt and non-prompt regime at some *critical mass*

$$M_{\rm crit} = \min(M) : \frac{dt_{\rm crit}}{dM_{\rm total}} < 0 \;\forall\; M_{\rm total} > M \,, \tag{5}$$

where $t_{\rm crit}(M_{\rm total})$ is the time at which an apparent horizon is formed.

- *Threshold masses of promptness p* characterize how strongly gravitational pull dominates over matter repulsion and overall collision dynamics

$$\left\{ M^{(p)}_{\rm th} = \min(M_{\rm total}) : \frac{d^p}{dt^p}\max(K_1) \geqslant 0 \;\forall\; t > t_{\rm merge} \right\} . \tag{6}$$



CE, Topolski, Järvinen, Stehr 2402.11013

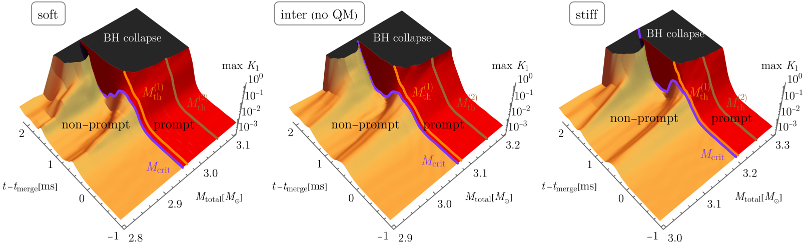
Scaling with the EOS Stiffness

- Soft to stiff EOS variants, i.e., small to large M_{TOV} and maximum c_s^2

$$2.95 M_\odot \lesssim M_{\mathrm{crit}} \lesssim 3.19 M_\odot\,. \quad (7)$$

- Consistent with non-prompt collapse outcome of GW170817 with $M_{\mathrm{total}}^{\mathrm{GW170817}} \approx 2.74\ M_\odot < M_{\mathrm{crit}}$, as suggested by observed EM emission.
- Rather tight mass-ratio:

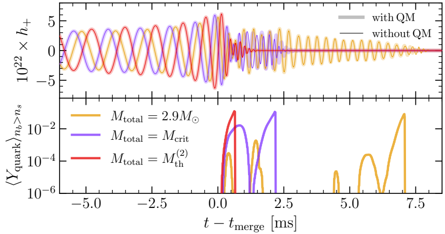
$$M_{\mathrm{crit}}/M_{\mathrm{TOV}} \approx 1.41 \pm 0.06 \quad (8)$$

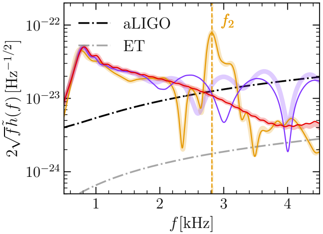


CE, Topolski, Järvinen, Stehr 2402.11013

Gravitational Waves

- Periodic recurrence of quark matter inside sub-critical HMNSs.
- Formation of sizeable quark-matter core induces collapse.
- Significant imprint of quark matter on GW spectrum only close to M_{crit}.
- Maybe EM counterpart is a more promising probe for quark matter?

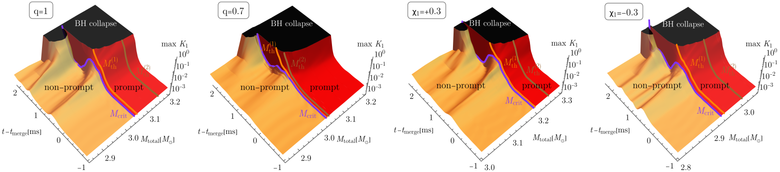




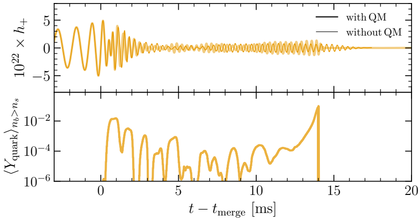
CE, Topolski, Järvinen, Stehr 2402.11013

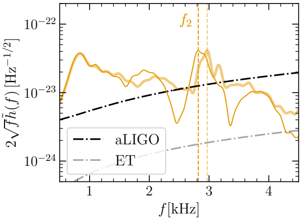
Impact of mass ratio and spin (preliminary)

- Small effect of mass asymmetry, but effect of spin can be large.



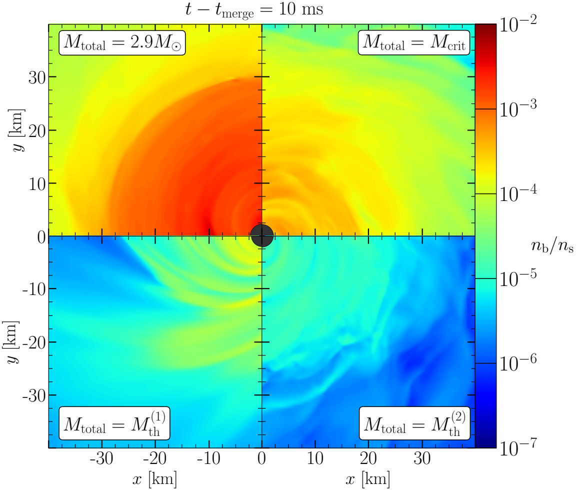
- Fine-tuned situation with "long-lived" quark matter core possible.





Residual Matter

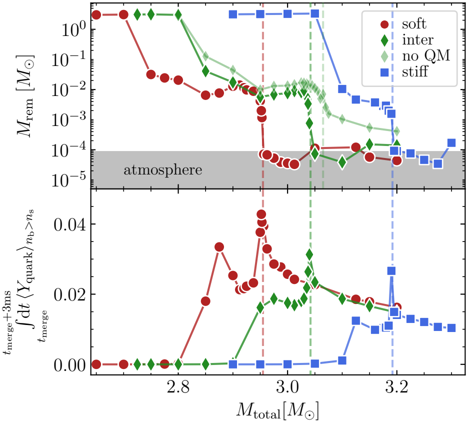
- EM counterpart needs matter to “shine”.
- Significant drop in residual matter outside horizon at $M_{\rm crit}$.



CE, Topolski, Järvinen, Stehr 2402.11013

Impact of Quark Matter

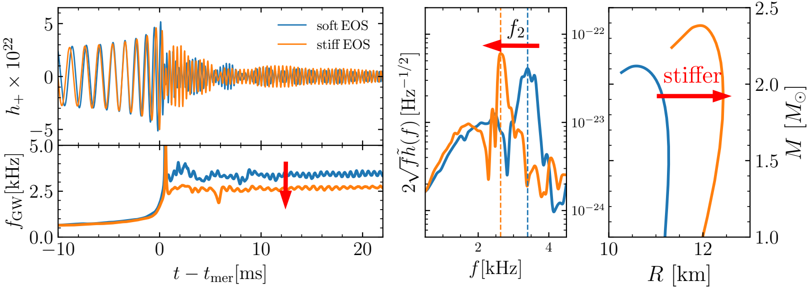
- **With quark matter:** essentially all matter is swallowed by the black hole in super-critical mergers ($M_{\rm total} > M_{\rm crit}$).
- Most quark matter formed in critical-mass mergers ($M_{\rm total} = M_{\rm crit}$).
- **Without quark matter:** some residual matter even for $M_{\rm total} > M_{\rm crit}$.
- Soft quark-matter cores collapse quicker and clear exterior more efficiently.



Listening to the long ringdown: a novel way to pinpoint the equation of state in neutron-star cores

Correlating Post-Merger GWs with EOS Properties

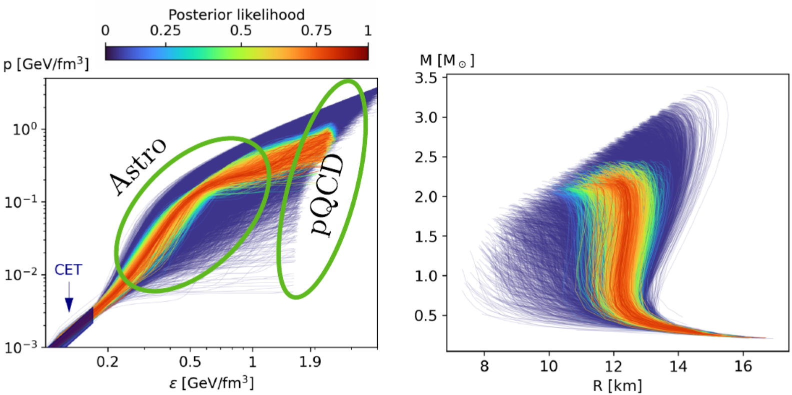
- Spectral features ($f_{\rm GW}$, f_2, etc.) of post-merger GWs correlate with EOS.
  many works: …; Takami, Rezzolla, Baiotti 1412.3240 (PRD); Bauswein, Stergioulas 1502.03176 (PRD); Rezzolla, Takami 1604.00246 (PRD); De Pietri et al 1910.04036 (PRD); Kiuchi et al 2211.07637 (PRL); Breschi et al 2110.06957 (PRL); …
- Mechanism: stiff EOS give large radii, slow rotation and low frequencies.
- Not observed so far, but expected to be seen by third-generation detectors: Cosmic Explorer + Einstein Telescope ≈ 180 BNSs/year (SNR>8).
  see, e.g., Evans et al 2109.09882 (Cosmic Explorer technical report)



Generic EOS Approach

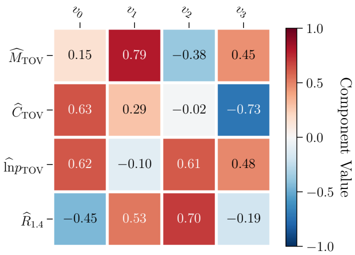
- So far: EOS-GW correlation studies use few "traditional" EOS models.
- However, generic EOS parametrizations reveal large freedom.
- Idea: Large ensemble ($> 10^5$) of generic EOSs that are constrained by astro, nuclear theory and perturbative QCD and cover allowed space.

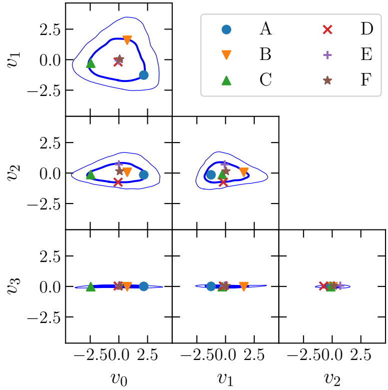
Gorda, Komoltsev, Kurkela 2204.11877 (ApJ)



Golden EOS Selection

- Too many possibilities to simulate: smart selection recipe is needed.
- Three variables ($M_{\rm TOV}, \mathcal{C}_{\rm TOV}, \ln p_{\rm TOV}$) to characterize the high-density part of the EOS and one ($R_{1.4}$) to break degeneracy at low densities.
- Principle component analysis: 4D distribution essentially 3D-triangular.
- Six "golden EOSs": A-E at corners of 68% contours and F in centre.

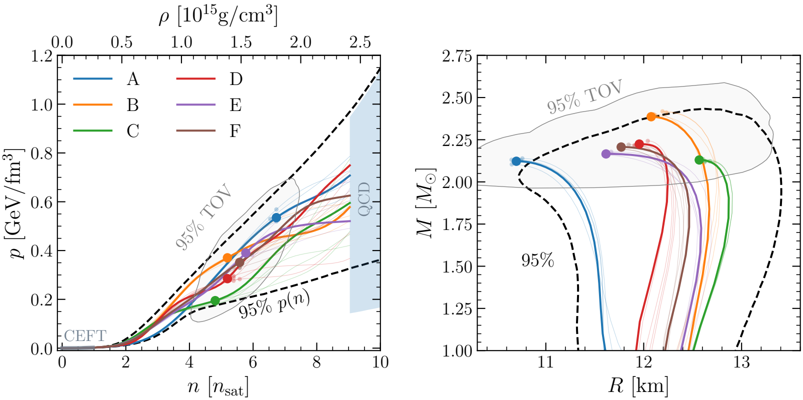




Ecker, Gorda, Kurkela, Rezzolla 2204.11877

Golden EOSs and Mass-Radius Relations

- Six EOS models manageable, but BNS parameter space still huge: fix $\mathcal{M}_{\rm chirp} = 1.18\ M_\odot$, but three different mass-ratios $q = \frac{M_1}{M_2} = 1, 0.85, 0.7$.
- Add T-dependence via simple gamma-law approximation with fixed $\Gamma_{\rm th} = 1.75$, but analysis remains robust when changing to $\Gamma_{\rm th} = 1.5, 2.0$.



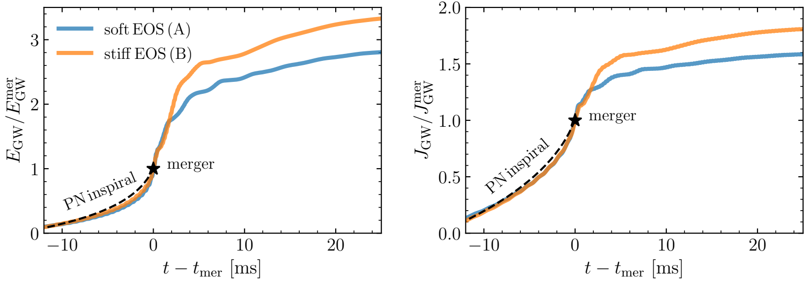
Emitted GW Energy and Angular Momentum

- Time evolution of GW strain components $h_+, h_\times$ from simulation.
- Emitted energy and angular momentum from post-processing

$$E_{\rm GW}(t) = \frac{r^2}{16\pi}\int_{-\infty}^{t} dt' \left(\dot{h}_+^2 + \dot{h}_\times^2\right) \,, \; J_{\rm GW}(t) = \frac{r^2}{16\pi}\int_{-\infty}^{t} dt' \left(h_+\dot{h}_\times - \dot{h}_+ h_\times\right) \,.$$

Bishop, Rezzolla 1606.02532 (Living Reviews in Relativity)

- Useful to normalise with $E_{\rm GW}^{\rm mer} := E_{\rm GW}(t_{\rm mer})$ and $J_{\rm GW}^{\rm mer} := J_{\rm GW}(t_{\rm mer})$.



5th-order post-Newtonian Taylor-T2 mode with $\tilde{\Lambda} = 580$ of PyCBC library, Biwer et al 1807.10312 (Publ.Astron.Soc.Pac.)

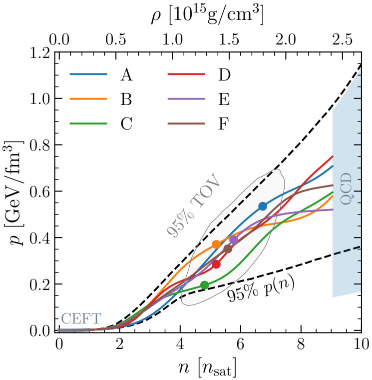
Long Ringdown

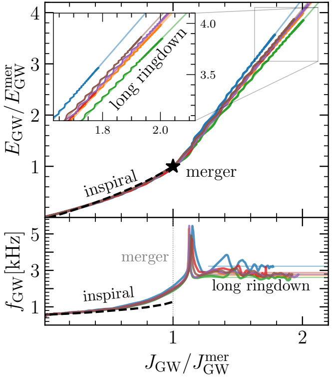
- Post-merger period where $E_{\rm GW}(t)$ and $J_{\rm GW}(t)$ are linearly related.
- Long-ringdown slope numerically close to GW frequency

$$\frac{dE_{\rm GW}}{dJ_{\rm GW}} = \frac{\dot{E}_{\rm GW}}{\dot{J}_{\rm GW}} = \frac{\dot{h}_+^2 + \dot{h}_\times^2}{h_+\dot{h}_\times - \dot{h}_+ h_\times}\,, \qquad f_{\rm GW} = \frac{1}{2\pi}\frac{h_+\dot{h}_\times - \dot{h}_+ h_\times}{h_+^2 + h_\times^2}\,.$$

- Identity for simple quadrupole system with $\ell = 2, m = 2$ deformation:

$$h_+(t) \propto \cos(\phi(t))\,, \quad h_\times(t) \propto \sin(\phi(t))\,, \quad \dot{E}_{\rm GW}/\dot{J}_{\rm GW} = f_{\rm GW}/(2\pi)\,.$$

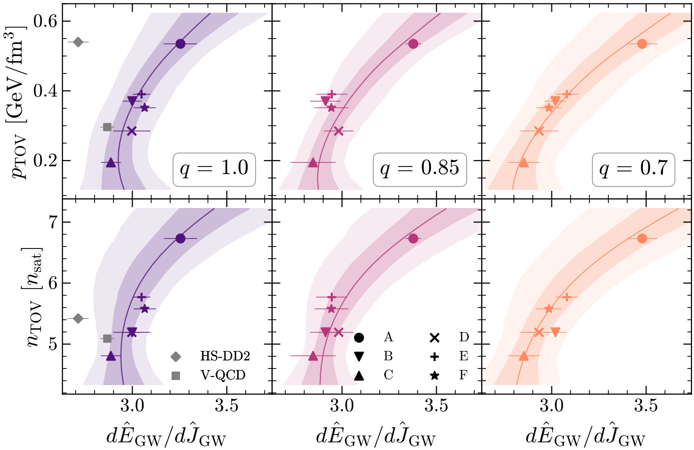




Correlations

- Slope correlates with maximum neutron star pressure and number density.
- Bilinear fit of simulation data constrains EOS at highest (TOV) densities

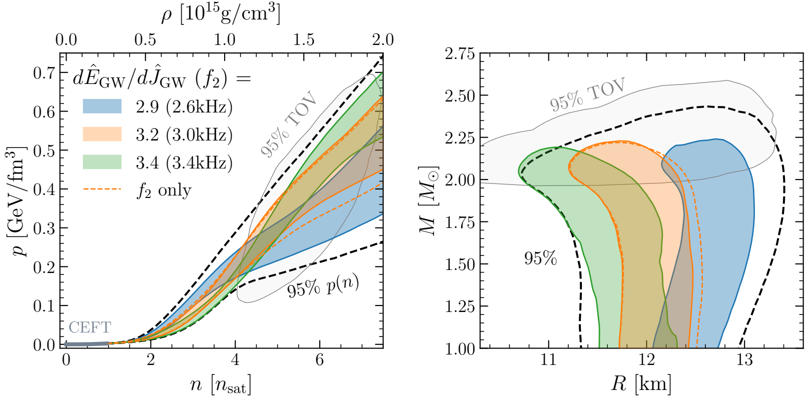
$$\frac{d\hat{E}_{\mathrm{GW}}}{d\hat{J}_{\mathrm{GW}}} = \beta_0 + \beta_1\, p_{\mathrm{TOV}} + \beta_2\, n_{\mathrm{TOV}} + \beta_3\, q + \beta_4\, q\, p_{\mathrm{TOV}} + \beta_5\, q\, n_{\mathrm{TOV}} + \beta_6\, p_{\mathrm{TOV}}\, n_{\mathrm{TOV}} \,.$$



Impact of Slope Measurement

- Assume measured slope $d\hat{E}_{\mathrm{GW}}/d\hat{J}_{\mathrm{GW}}$ and f_2 with $\pm 4\%$ uncertainty.
- Correlation gives new likelihood to update EOS constraints at all densities.
- Slightly improvement compared to measuring just f_2.
- Pearson-correlation coefficients slightly larger for $d\hat{E}_{\mathrm{GW}}/d\hat{J}_{\mathrm{GW}}$ than for f_2

$$r(dE_{\mathrm{GW}}/dJ_{\mathrm{GW}}, p_{\mathrm{TOV}}) = 0.877 \quad \text{vs} \quad r(f_2, p_{\mathrm{TOV}}) = 0.792\,.$$



Summary

Prompt Collapse:

- Hot V-QCD hybrid EOS with deconfinement phase transition, three tabulated versions on CompOSE database: DEJ(DD2-VQCD).
- Three different stages in HMNSs: hot, warm, cold quark matter.
- Classification of prompt collapse with curvature invariants: $M_{\rm crit}$,$M_{\rm th}^{(p)}$.
- Relation between critical mass and TOV-mass: $M_{\rm crit}/M_{\rm TOV} \approx 1.41 \pm 0.06$
- Most quark matter formed in critical-mass mergers: $M_{\rm total} = M_{\rm crit}$.
- Small impact on GW spectrum, but maybe visible from EM counterpart.

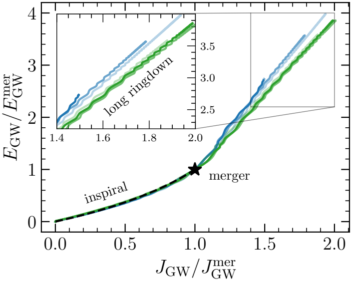
Long Ringdown:

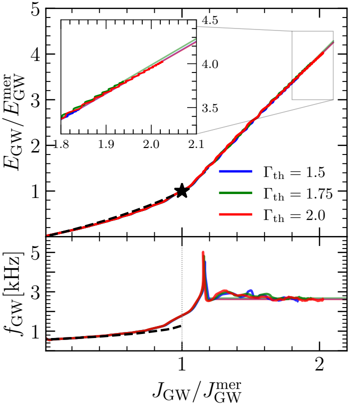
- Principle component analysis to single out few golden EOSs.
- Novel correlations between long ringdown slope $d\hat{E}_{\rm GW}/d\hat{J}_{\rm GW}$ and $(p_{\rm TOV}, n_{\rm TOV})$ constrain the EOS at maximum NS core densities.
- Correlation + Bayes theorem: EOS (MR) constraints at all densities.

Backup

Impact of Chirp Mass and Temperature

- Dependence on chirp mass much smaller than EOS dependence.
- Slope is essentially insensitive to finite temperature effects.





Robustness

- Surprisingly insensitive to grid-resolution: even extremely coarse (cheap) and fine (expensive) grids give almost identical values for the slope.
- Reason: post-merger GW dominated by global $\ell = m = 2$ deformations of merger-remnant which are only weakly influenced by small scale effects.