

Listening to the long ringdown: a novel way to pinpoint the EOS in neutron-star cores

Christian Ecker

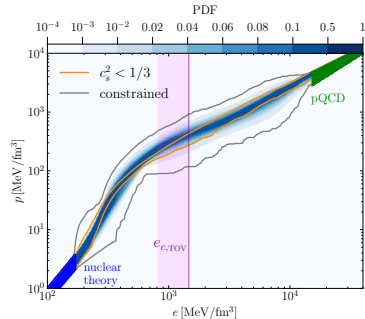
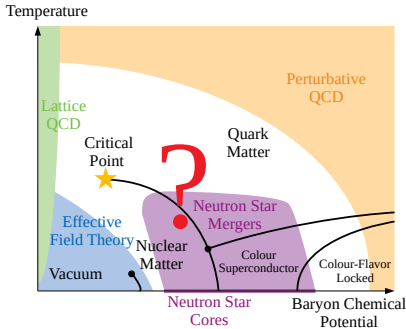


Quark Matter 2025
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Based on Nature Commun. 16 (2025) 1, 1320 (2403.03246) with
Tyler Gorda, Aleksi Kurkela and Luciano Rezzolla

Motivation

- ▶ **Goal:** Infer QCD phase diagram and the equation of state (EOS).
- ▶ **Approach:** Synthesis of EOS theory, simulation and observation.
- ▶ **This talk:** Using relation between gravitational wave energy and angular momentum (long-ringdown slope) as observable.

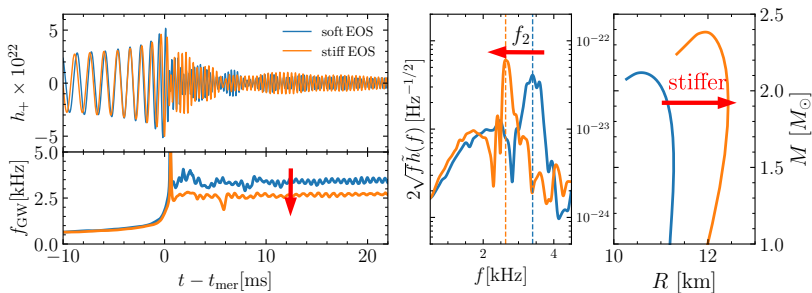


plot: Altiparmak, CE, Rezzolla 2203.14974 (ApJL)

Lots of recent work: ... ; Annala, Gorda, Kurkela, Nättilä, Vuorinen 2105.05132 (PRX); Komoltsev, Kurkela 2111.05350 (PRL); Gorda, Komoltsev, Kurkela 2204.11877 (ApJ); CE, Rezzolla 2207.04417 (ApJL); Brandes, Weise, Kaiser 2208.03026 (PRD); ...

Correlating Post-Merger GWs with EOS Properties

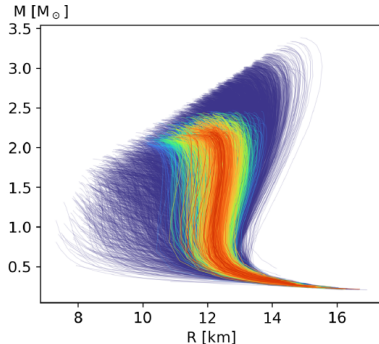
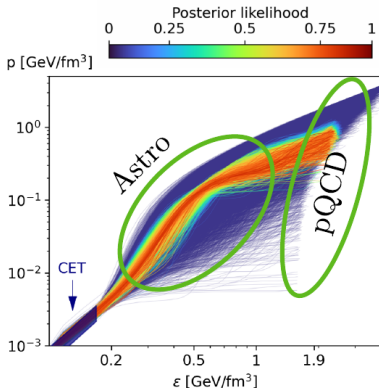
- ▶ Spectral features (f_{GW} , f_2 , etc.) of post-merger GWs correlate with EOS.
many works: ...; Takami, Rezzolla, Baiotti 1412.3240 (PRD); Bauswein, Stergioulas 1502.03176 (PRD); Rezzolla, Takami 1604.00246 (PRD); De Pietri et al 1910.04036 (PRD);
Kiuchi et al 2211.07637 (PRL); Breschi et al 2110.06957 (PRL); ...
- ▶ Mechanism: stiff EOS give large radii, slow rotation and low frequencies.
- ▶ Not observed so far, but expected to be seen by third-generation detectors: Cosmic Explorer + Einstein Telescope ≈ 180 BNSs/year (SNR >8).
see, e.g., Evans et al 2109.09882 (Cosmic Explorer technical report)



Generic EOS Approach

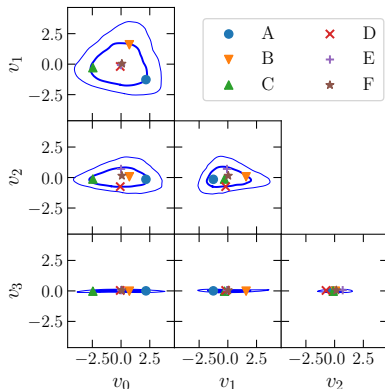
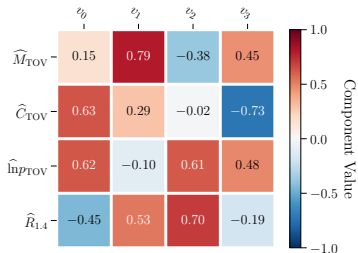
- ▶ So far: EOS-GW correlation studies use few “traditional” EOS models.
- ▶ However, generic EOS parametrizations reveal large freedom.
- ▶ Idea: Large ensemble ($> 10^5$) of generic EOSs that are constrained by astro, nuclear theory and perturbative QCD and cover allowed space.
- ▶ EOS ensemble: Gaussian process regression method conditioned with Bayesian likelihoods from dense matter theory and astro measurements.

Gorda, Komoltsev, Kurkela 2204.11877 (ApJ)



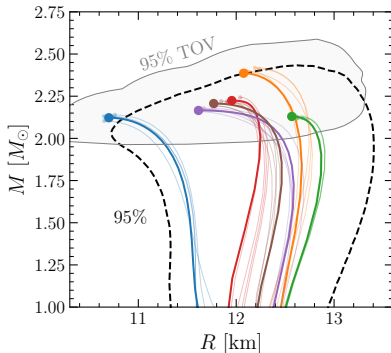
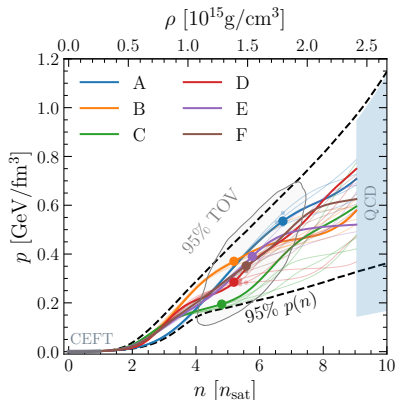
Golden EOS Selection

- ▶ Too many possibilities to simulate: smart selection recipe is needed.
- ▶ Three variables (M_{TOV} , C_{TOV} , $\ln p_{\text{TOV}}$) to characterize the high-density part of the EOS and one ($R_{1.4}$) to break degeneracy at low densities.
- ▶ Principle component analysis: 4D distribution essentially 3D-triangular.
- ▶ Six “golden EOSs”: A-E at corners of 68% contours and F in centre.



Golden EOSs and Mass-Radius Relations

- ▶ Six EOS models manageable, but BNS parameter space still huge: fix $\mathcal{M}_{\text{chirp}} = 1.18 M_{\odot}$, but three different mass-ratios $q = \frac{M_1}{M_2} = 1, 0.85, 0.7$.
- ▶ Add T -dependence via simple gamma-law approximation with fixed $\Gamma_{\text{th}} = 1.75$, but analysis remains robust when changing to $\Gamma_{\text{th}} = 1.5, 2.0$.



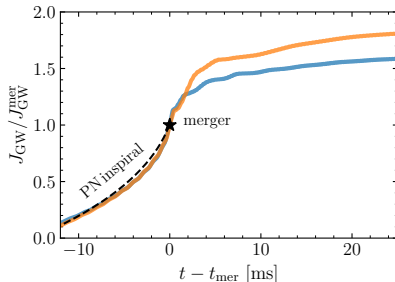
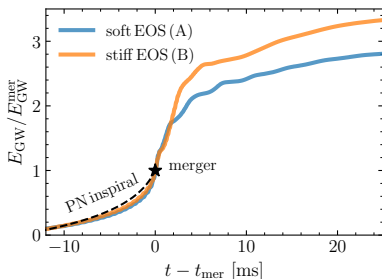
Emitted GW Energy and Angular Momentum

- ▶ Time evolution of GW strain components h_+ , $h_\times$ from simulation.
- ▶ Emitted energy and angular momentum from post-processing

$$E_{\text{GW}}(t) = \frac{r^2}{16\pi} \int_{-\infty}^t dt' \left(\dot{h}_+^2 + \dot{h}_\times^2 \right), \quad J_{\text{GW}}(t) = \frac{r^2}{16\pi} \int_{-\infty}^t dt' \left(h_+ \dot{h}_\times - \dot{h}_+ h_\times \right).$$

see, e.g., Bishop, Rezzolla 1606.02532 (Living Reviews in Relativity)

- ▶ Useful to normalise with $E_{\text{GW}}^{\text{mer}} := E_{\text{GW}}(t_{\text{mer}})$ and $J_{\text{GW}}^{\text{mer}} := J_{\text{GW}}(t_{\text{mer}})$.



5th-order post-Newtonian Taylor-T2 mode with $\tilde{\Lambda} = 580$ of PyCBC library, Biwer et al 1807.10312 (Publ.Astron.Soc.Pac.)

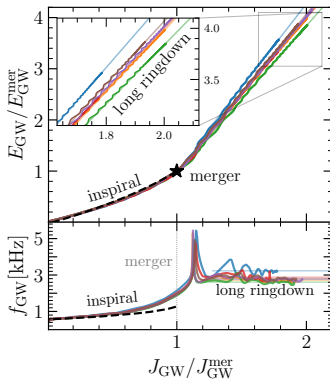
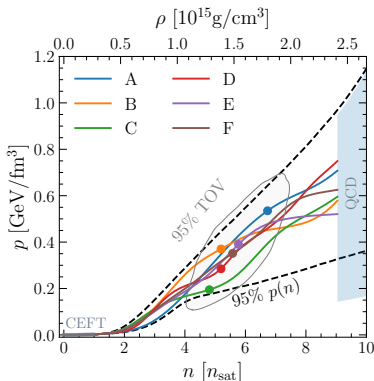
Long Ringdown

- Post-merger period where $E_{\text{GW}}(t)$ and $J_{\text{GW}}(t)$ are linearly related.
- Long-ringdown slope numerically close to GW frequency

$$\frac{dE_{\text{GW}}}{dJ_{\text{GW}}} = \frac{\dot{E}_{\text{GW}}}{\dot{J}_{\text{GW}}} = \frac{\dot{h}_+^2 + \dot{h}_\times^2}{h_+ \dot{h}_\times - \dot{h}_+ h_\times}, \quad f_{\text{GW}} = \frac{1}{2\pi} \frac{h_+ \dot{h}_\times - \dot{h}_+ h_\times}{h_+^2 + h_\times^2}.$$

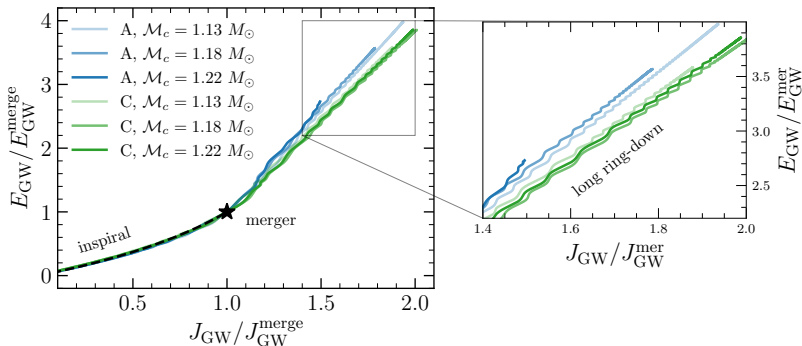
- Identity for simple quadrupole system with $\ell = 2, m = 2$ deformation:

$$h_+(t) \propto \cos(\phi(t)), \quad h_\times(t) \propto \sin(\phi(t)), \quad \dot{E}_{\text{GW}}/\dot{J}_{\text{GW}} = f_{\text{GW}}/(2\pi).$$



Robustness

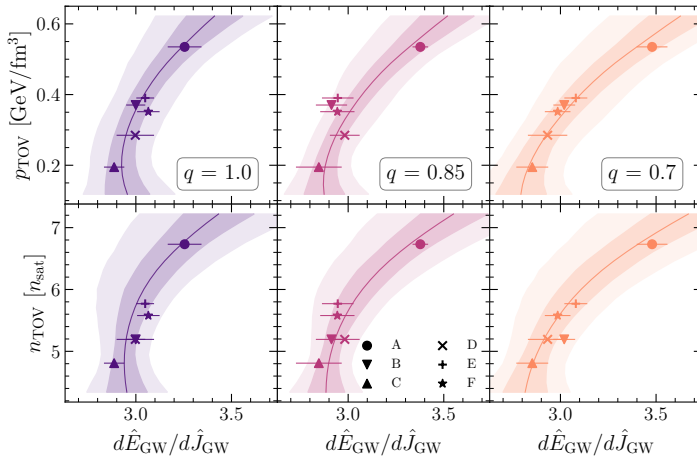
- ▶ Waveform dominated by the large-scale deformations of post-merger remnant, only weak influence of small-scale features.
- ▶ Essentially no difference between simulations with 200 – 400 m resolution.
- ▶ Also insensitive to approximate thermal effects for $\Gamma_{\text{th}} = 1.5 - 2$.
- ▶ However, sensitive to BNS parameters such as chirp mass, but sufficiently different EOSs remain distinguishable.



Correlations

- ▶ Slope correlates with maximum neutron star pressure and number density.
- ▶ Bilinear fit of simulation data constrains EOS at highest (TOV) densities

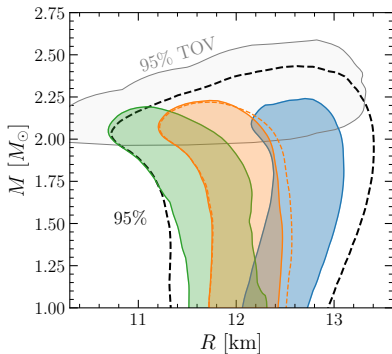
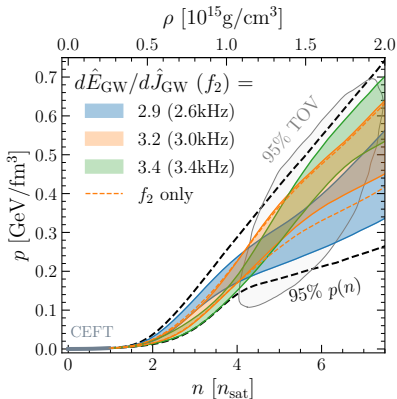
$$\frac{d\hat{E}_{\text{GW}}}{d\hat{J}_{\text{GW}}} = \beta_0 + \beta_1 p_{\text{TOV}} + \beta_2 n_{\text{TOV}} + \beta_3 q + \beta_4 q p_{\text{TOV}} + \beta_5 q n_{\text{TOV}} + \beta_6 p_{\text{TOV}} n_{\text{TOV}} .$$



Impact of Slope Measurement

- ▶ Mock measurement: assume measured values for the slope $d\hat{E}_{\text{GW}}/d\hat{J}_{\text{GW}}$ and f_2 with $\pm 4\%$ measurement uncertainty.
- ▶ Correlation gives new evidence to update EOS constraints at all densities.
- ▶ Slightly improvement compared to measuring just f_2 .
- ▶ Pearson-correlation coefficients slightly larger for $d\hat{E}_{\text{GW}}/d\hat{J}_{\text{GW}}$ than for f_2

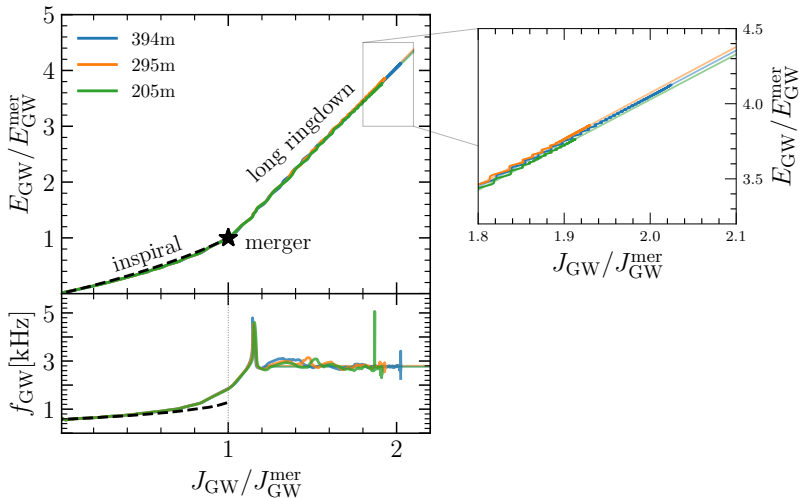
$$r(dE_{\text{GW}}/dJ_{\text{GW}}, p_{\text{TOV}}) = 0.877 \quad \text{vs} \quad r(f_2, p_{\text{TOV}}) = 0.792.$$



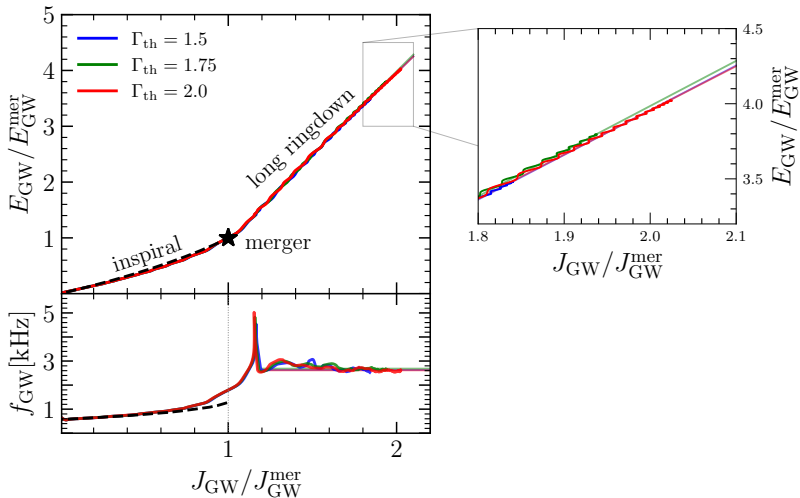
Summary

- ▶ Third-generation GW observatories are expected to see numerous BNS post-merger waveforms at high SNR.
- ▶ Looking for EOS-GW correlations using large generic EOS set.
- ▶ Principle component analysis to single out few golden EOSs.
- ▶ Novel correlations between long ringdown slope $d\hat{E}_{\text{GW}}/d\hat{J}_{\text{GW}}$ and $(\rho_{\text{TOV}}, n_{\text{TOV}})$ constrain the EOS at maximum NS core densities.
- ▶ Slope $d\hat{E}_{\text{GW}}/d\hat{J}_{\text{GW}}$ simple to compute from the waveform.
- ▶ Correlation + Bayes theorem: EOS (MR) constraints at all densities.
- ▶ Improvement compared to measuring f_2 only.

Impact of Resolution



Impact of Gamma



Tabulated Models

