# A dynamical inflaton coupled to strongly interacting matter

Christian Ecker

Theory Seminar, TU-Wien
25 May 2023

Based on 2302.06618 (PRL)
with
Elias Kiritsis and Wilke van der Schee

# Motivation

- Precise measurements of the temperature anisotropies in the cosmic microwave background have revealed that our universe is homogeneous, isotropic and almost flat on large scales.
- Compelling evidence for inflation: period of exponential expansion of space in the early universe that provides a natural solution for the horizon, flatness and magnetic-monopole problem in big-bang cosmology.

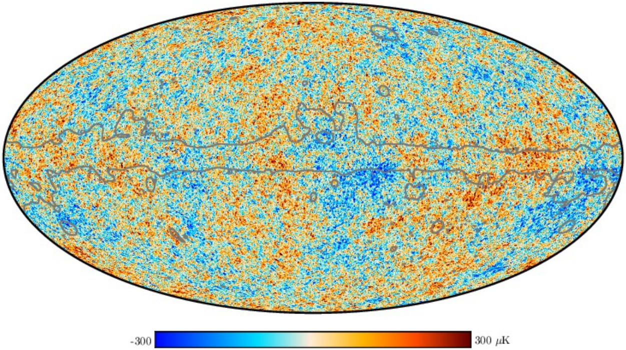
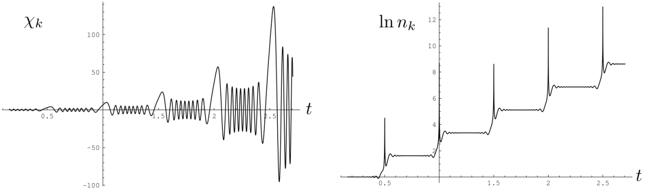



Image: Planck Collaboration arXiv:1807.06205 (Astronomy & Astrophysics)

# Reheating the Universe

- After the end of inflation the universe is large, empty and cold. A mechanism that creates the matter content observed today is needed.
- Energy scale at end of inflation can be as high as $\sim 10^{16}$ GeV where the universe is only $\sim 10^{-36}$ s old.
- Primordial nucleosynthesis of light elements happens at a scale of $\sim 1$ MeV when the universe is older than $\gtrsim 1$ s, nuclear theory predictions can be compared to abundances observed in the current universe.
- Huge range in energy and time that is poorly constrained by observations.
- Reheating is the energy transfer of the inflaton field to the standard model content of the universe during this poorly understood period.
- Traditional: perturbation theory of inflaton decay ($\phi \to \chi\chi$), resonant Bose enhancement leads to exponential energy transfer (preheating).




See, e.g., Kofman, Linde, Starobinsky arXiv:9704452 (PRD), or lecture notes by Kaloian Lozanov arXiv:1907.04402 and references therein.

# Holographic Model for Inflationary Cosmology

**Desired features:**

- Early epoch of inflationary expansion and cooling.
- Mechanism that transfers inflaton energy to a holographic QFT.
- Thermalization of QFT matter in the late universe.

**Caveats:**

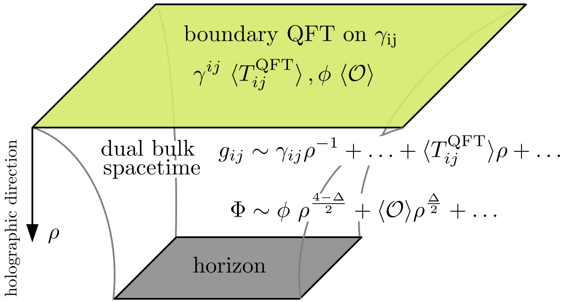
- Not unique: many possible choices for holographic theory, inflaton potential, coupling between different sectors, initial conditions, ...
- Holographic QFTs are strongly coupled: not QCD, hidden BSM sector?
- **This talk:** only a proof of principle of the method with a specific setup rather than a phenomenological model of our universe.

# AdS/CFT

Standard AdS/CFT assumes fixed boundary conditions $\gamma_{ij}$, $\phi$ (sources)

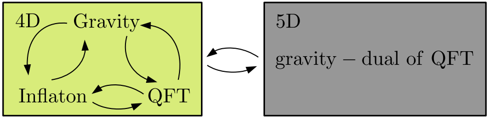
$$Z_{\mathrm{QFT}}[\gamma, \phi] = \int [\mathcal{D}g]_\gamma [\mathcal{D}\Phi]_\phi e^{-S_{\mathrm{hol}}[g,\Phi]} \leftrightarrow \mathcal{L}_{\mathrm{CFT}} + \Lambda^{4-\Delta}\mathcal{O}\,. \tag{1}$$

$$\langle T_{ij}^{\mathrm{QFT}} \rangle = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\mathrm{hol}}}{\delta \gamma^{ij}}\,, \quad \langle \mathcal{O} \rangle = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\mathrm{hol}}}{\delta \phi} \tag{2}$$



# Framework

**Goal:** Build a framework that is able to describe dynamically the mutual interaction between gravity, the inflaton and a strongly coupled holographic QFT in a self-consistent way.




**Minimal ingredients:** Einstein-Hilbert+Inflaton action, a holographic QFT and some interaction that couples them non-minimally

$$S = S_{\mathrm{EH+inf}} + S_{\mathrm{hol}} + S_{\mathrm{int}} \, . \tag{3}$$

Many possibilities for $S$: we choose a specific setup that is guided by simplicity and intuition about inflationary cosmology.

**Non-standard AdS/CFT:** Coupling bulk and boundary dynamics requires AdS/CFT with mixed boundary conditions.

Compere, Marolf arXiv:0805.1902 (Class.Quant.Grav)

# AdS/CFT with Dynamic Boundary Conditions

Promote boundary conditions to dynamical fields:

$$Z_{\text{ind}} = \int \mathcal{D}\gamma \mathcal{D}\phi Z_{\text{QFT}}[\gamma, \phi] = \int \mathcal{D}g \mathcal{D}\Phi e^{-S_{\text{hol}}[g,\Phi]}, \tag{4}$$

$$\delta_g S_{\text{hol}} = \int_{\mathcal{M}} \mathrm{d}x^5 \sqrt{-g}\, \text{EOM}^{(\text{g})}_{\text{bulk}}\, \delta g^{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}x^4 \sqrt{-\gamma} \frac{1}{2} \langle T^{\text{QFT}}_{ij} \rangle \delta\gamma^{ij}, \tag{5}$$

$$\delta_\Phi S_{\text{hol}} = \int_{\mathcal{M}} \mathrm{d}x^5 \sqrt{-g}\, \text{EOM}^{(\Phi)}_{\text{bulk}}\, \delta\Phi + \int_{\partial\mathcal{M}} \mathrm{d}x^4 \sqrt{-\gamma} \langle \mathcal{O} \rangle \delta\phi\,. \tag{6}$$

There are different ways to make the action stationary

- Dirichlet boundary conditions: $\delta\gamma_{ij} = 0$, $\delta\phi = 0$
- Neumann boundary conditions: $\langle T^{\text{QFT}}_{ij} \rangle = 0$, $\langle \mathcal{O} \rangle = 0$
- Mixed boundary conditions:

$$\frac{1}{2}\langle T^{\text{QFT}}_{ij} \rangle + \frac{\delta S_{\text{bdry},\gamma}}{\delta\gamma^{ij}} = 0\,, \quad \langle \mathcal{O} \rangle + \frac{\delta S_{\text{bdry},\phi}}{\delta\phi} = 0 \tag{7}$$

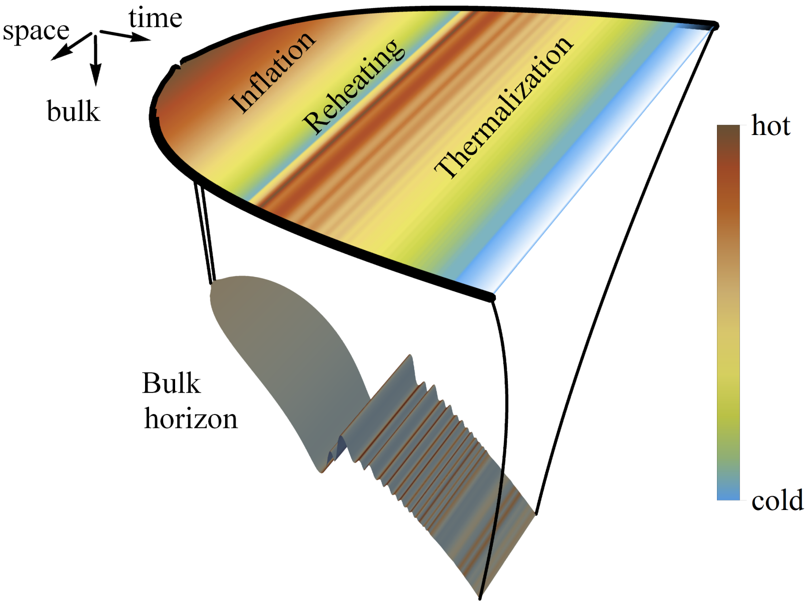
Compere, Marolf arXiv:0805.1902 (Class.Quant.Grav)

**This talk:** impose Einstein-Hilbert-inflaton action + interaction term

$$S_{\text{bdry},\gamma,\phi} = S_{\text{EH+inf}}[\gamma_{ij}, \phi] + S_{\text{int}}[\gamma_{ij}, \phi]\,. \tag{8}$$

Generalization of previous works with dynamical boundary gravity (without inflaton) and for semi-holographic glasma (class. Yang-Mills) evolution (without gravity).

CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2011.08194 (JHEP)
CE, Mukhopadhyay, Preis, Rebhan, Soloviev arXiv:1806.01850 (JHEP)


space
time
bulk
Inflation
Reheating
Thermalization
hot
cold
Bulk
horizon

# Boundary Gravity and Inflaton

Einstein-Hilbert+Inflaton action

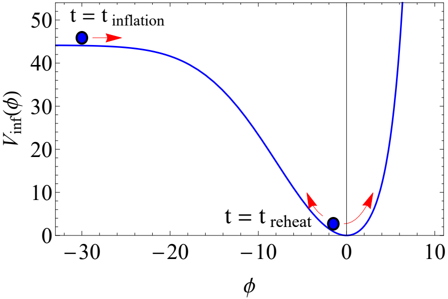
$$S_{\mathrm{EH+inf}} = \int \mathrm{d}^4 x \sqrt{-\gamma} \left( \frac{R}{2\kappa_4} - \frac{1}{2}\gamma^{ij}\partial_i\phi\partial_j\phi - V_{\mathrm{inf}}(\phi) \right) . \tag{9}$$

Friedmann–Lemaître–Robertson–Walker type metric

$$\mathrm{d}s^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2 . \tag{10}$$

Coleman-Weinberg type inflaton potential with fixed $v_0$ and $\phi_m$ such that $V_{\mathrm{inf}}(0) = V'_{\mathrm{inf}}(0) = 0$

$$V_{\mathrm{inf}}(\phi) = v_0 + \tfrac{9}{8}\, e^{\frac{2}{3}(\phi-\phi_m)} - 45\, e^{-\frac{1}{50}(\phi-\phi_m)^2} . \tag{11}$$



# Holographic QFT

5D Einstein-dilaton gravity with non-trivial potential

$$S_{\rm bulk} = \frac{2}{\kappa_5}\int {\rm d}^5x\sqrt{-g}\left(\frac{1}{4}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V_{\rm bulk}(\Phi)\right) . \qquad (12)$$

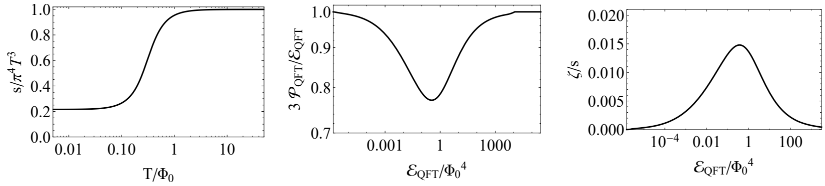
Potential inspired by GPPZ model: holographic SUGRA dual of $\mathcal{N} = 1$ SYM

Girardello, Petrini, Porrati, Zaffaroni, arXiv:hep-th/9909047 (Nucl.Phys. B)

$$V_{\rm bulk}(\Phi) = \frac{1}{L^2}\left(-3 - \frac{3\Phi^2}{2} - \frac{\Phi^4}{3} + \frac{11\Phi^6}{96} - \frac{\Phi^8}{192}\right) . \qquad (13)$$

Smooth RG-flow between IR and UV fixed points and broken conformal symmetry between.

Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Olivan, Sopuerta, Triana, Zilhao arXiv:1603.01254 (JHEP)



# Fixing Ambiguities

The bare bulk action $S_{\rm bulk}$ needs to be remormalized by adding appropriate counter-terms that render the holographic action $S_{\rm hol}$ finite on-shell

$$S_{\rm hol} = S_{\rm bulk} + S_{\rm ct}\,. \tag{14}$$

Counter terms are unique up to some finite contributions

$$S_{\rm ct} = \frac{1}{\kappa_5}\int {\rm d}^4x\sqrt{-\gamma}\left[\left(-\frac{1}{8}R - \frac{3}{2} - \frac{1}{2}\Phi_{(0)}^2\right) + \frac{1}{2}\left(\log\rho\right)\mathcal{A} + \left(\alpha\mathcal{A} + \beta\Phi_{(0)}^4\right)\right]. \tag{15}$$

Anomaly due to curved boundary metric and broken conformal symmetry

$$\mathcal{A} = \frac{1}{16}(R^{ij}R_{ij} - \frac{1}{3}R^2) - \frac{1}{2}\left(\partial_i\Phi_{(0)}\partial^i\Phi_{(0)} + \frac{1}{6}R\Phi_{(0)}^2\right)\,. \tag{16}$$

Arbitrary constants $\alpha$ and $\beta$ renormalize the bare gravitational coupling and the cosmological constant in the boundary theory

$$\frac{1}{\kappa_4} = \frac{1}{\kappa_{4,\rm bare}} + \frac{\alpha}{96\,\kappa_5}\,, \tag{17}$$

$$\frac{\Lambda_4}{\kappa_4} = \frac{\Lambda_{4,\rm bare}}{\kappa_4} - \frac{\beta}{1024\,\pi}\,. \tag{18}$$

We choose the "supersymmetric" renormalization scheme $\alpha = 0$, $\beta = \frac{1}{16}$.

CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)

# Interaction Term

The interaction term couples the inflaton via the vacuum expectation value (VEV) of the scalar operator $\langle \mathcal{O} \rangle = \mathcal{O}_{\mathrm{QFT}}/\kappa_5$ to the holographic sector

$$S_{\mathrm{int}} = \int \mathrm{d}^4 x \sqrt{-\gamma}\; U(\phi)\; \mathcal{O}_{\mathrm{QFT}} \,. \tag{19}$$

In the QFT the inflaton hence acts as a source for the scalar operator $\mathcal{O}$, where the source is given by the boundary value of the bulk scalar $\Phi_{(0)} = U(\phi)$.

Different choices for $U(\phi)$ are possible, but in the following we assume linear coupling for simplicity

$$U(\phi) = \lambda\phi \,, \quad \lambda = \mathrm{const.} \,. \tag{20}$$

# Energy-Momentum Tensor

The total EMT in the boundary theory consists of three parts

$$T_{ij} = T_{ij}^{\rm inf} + \mathcal{T}_{ij}^{\rm QFT} + T_{ij}^{\rm int} = {\rm diag}\left(\mathcal{E}, \mathcal{P}, \mathcal{P}, \mathcal{P}\right) . \qquad (21)$$

Inflaton contribution is given by the standard scalar field expression

$$T_{ij}^{\rm inf} = \partial_i \phi \partial_j \phi - \gamma_{ij} \left( \frac{1}{2} \partial_k \phi \partial^k \phi + V_{\rm inf} \right) . \qquad (22)$$

Holographic part follows from variation of the renormalized holography action

$$\mathcal{T}_{ij}^{\rm QFT} = \frac{2}{\kappa_5 \sqrt{-\gamma}} \frac{\delta S_{\rm hol}}{\delta \gamma^{ij}} = {\rm diag}\left(\mathcal{E}_{\rm QFT}, \mathcal{P}_{\rm QFT}, \mathcal{P}_{\rm QFT}, \mathcal{P}_{\rm QFT}\right) \qquad (23)$$

EMT and scalar VEV are related by anomaly corrected Ward identities

$$\gamma^{ij} \mathcal{T}_{ij}^{\rm QFT} = \mathcal{E}_{\rm QFT} - 3\, \mathcal{P}_{\rm QFT} = -U(\phi)\, \mathcal{O}_{\rm QFT} + \mathcal{A} , \qquad (24)$$

$$\nabla^i \mathcal{T}_{ij}^{\rm QFT} = -\partial_j\, U(\phi)\, \mathcal{O}_{\rm QFT} . \qquad (25)$$

The direct coupling between the inflaton and the holographic sector gives

$$T_{ij}^{\rm int} = -\frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\rm int}}{\delta \gamma^{ij}} = U(\phi)\, \mathcal{O}_{\rm QFT}\, \gamma_{ij} . \qquad (26)$$

The total EMT is covariantly conserved on-shell: $\nabla_i T^{ij} = 0$.

# Initial Value Problem

**4D** Friedmann + scalar field equation for the inflaton that is coupled to $\mathcal{O}_{\rm QFT}$

$$H(t)^2 = \frac{\kappa_4}{3}\mathcal{E}(t)\,, \tag{27}$$

$$\frac{a''(t)}{a(t)} = -\frac{1}{2}\left(\kappa_4\mathcal{P}(t) + H(t)^2\right)\,, \tag{28}$$

$$\phi''(t) = \partial_\phi\, U(\phi(t))\,\mathcal{O}_{\rm QFT}(t) - 3H(t)\phi'(t) - \partial_\phi\, V_{\rm inf}(\phi(t))\,, \tag{29}$$

where $H(t) = a'(t)/a(t)$ is the Hubble rate.

**5D** Einstein–Klein–Gordon equations

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\, g_{\mu\nu} = 2\partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(2V_{\rm bulk} + (\partial\Phi)^2\right)\,, \tag{30}$$

$$\Box_g\Phi = \frac{\partial V_{\rm bulk}}{\partial\Phi}\,, \tag{31}$$

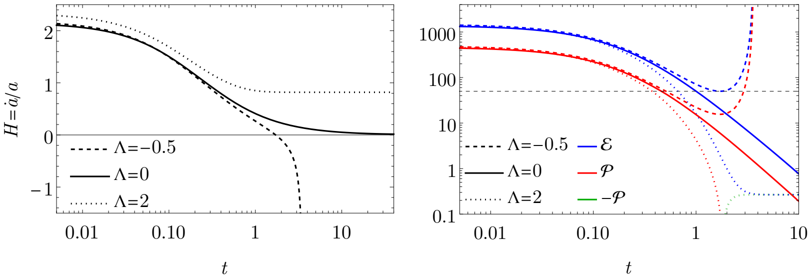
with Friedmann–Lemaitre–Robertson–Walker metric imposed at the boundary

$$\mathrm{d}s^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2\,. \tag{32}$$

Coupled system has to be solved numerically with appropriate parameters $(\kappa_5, \kappa_4, \lambda)$ and initial conditions $(\mathcal{E}_{\rm QFT}^{\rm ini}, \phi_{\rm ini}, \phi'_{\rm ini}, \Phi_{\rm ini}(r))$ to get the time evolution of the scale factor $a(t)$, the inflaton $\phi(t)$ and the EMT $T_{ij}(t)$.

# Static Inflaton

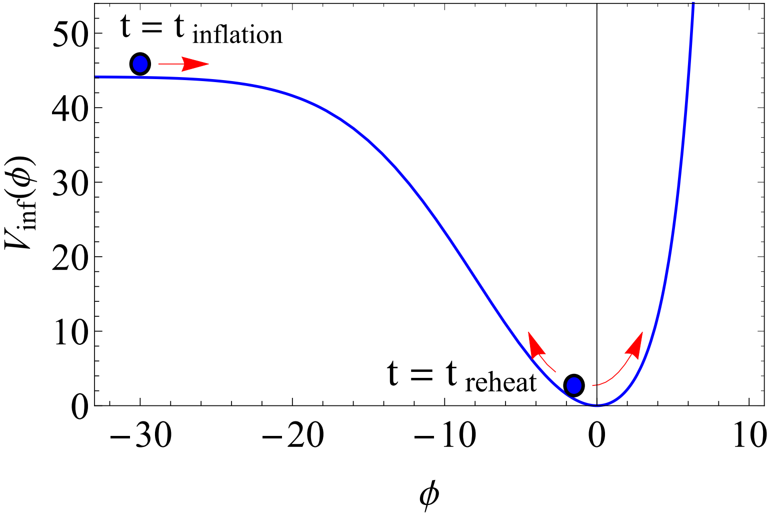
- Let's first see what happens when we freeze the inflaton: $\phi(t) = \text{const.}$.
- Pick some value for cosmological constant Λ and initialize QFT with equilibrium state on flat-space $=$ excited state on curved background.
- Depending on the value of Λ, the universe ends up in a big crunch (Λ $<$ 0), in flat-space (Λ $=$ 0) or in de Sitter (Λ $>$ 0).




CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)

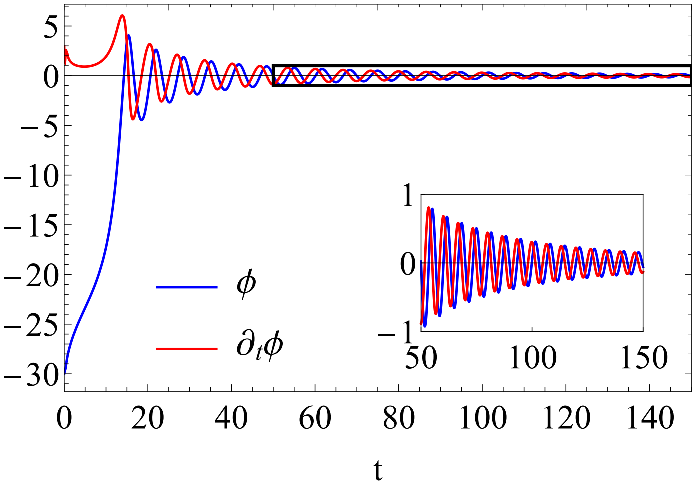
# Dynamic Inflaton

- Initially slow-rolling inflaton induces exponentially expanding universe.
- Inflation ends when the inflaton rolls into the bottom of the potential.
- Oscillations around the minimum reheat the QFT.



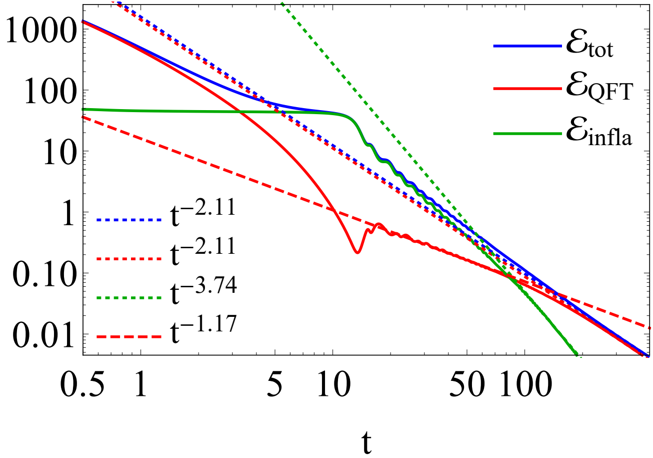
# Inflaton

- Early phase ($t \lesssim 3$) dominated by QFT energy, after which the universe enters a period of constant exponential expansion.
- At arround $t \approx 14$ the inflaton reaches the bottom of the potential where it oscillates rapidly and sources the energy for the QFT.



# Energy Density

- QFT energy is dominant until $t \approx 3$, then the inflaton dominates and until it reaches the bottom of the potential ($t \approx 14$).
- Inflaton oscillations reheat the QFT from $\mathcal{E}_{\mathrm{QFT}} = 0.21$ at $t = 13.5$ to a subsequent maximum of $\mathcal{E}_{\mathrm{QFT}} = 0.64$ at $t = 17.3$.
- Reheating continues: relatively slow scaling $\mathcal{E}_{\mathrm{QFT}} \propto t^{-1.17}$ of the QFT

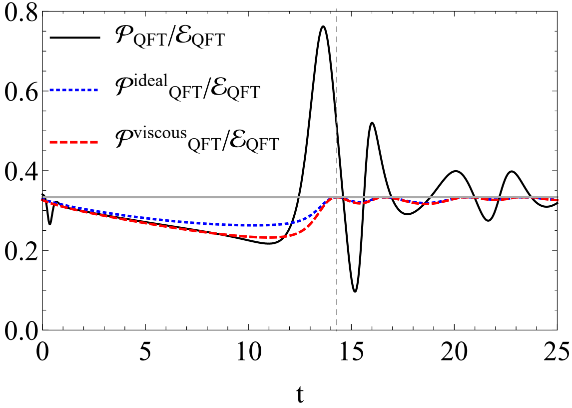


# QFT Pressure

- After a short far-from-equilibrium stage, the system is well described by hydrodynamics until the inflaton sources the QFT out of equilibrium

$$\mathcal{P}^{\mathrm{viscous}}_{\mathrm{QFT}}(t) = \mathcal{P}^{\mathrm{ideal}}_{\mathrm{QFT}}(\mathcal{E}_{\mathrm{QFT}}(t)) - 3H\zeta(\mathcal{E}_{\mathrm{QFT}}(t)) + \mathcal{O}(H^2)\,. \tag{33}$$

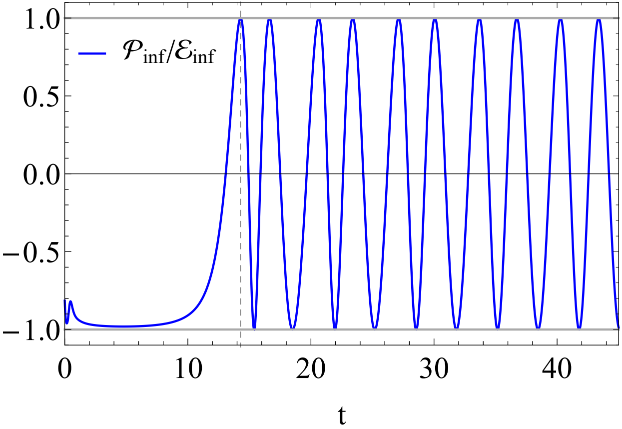
- QFT evolves from the UV to the IR fixed point where $\mathcal{P}_{\mathrm{QFT}} = \mathcal{E}_{\mathrm{QFT}}/3$.




as also seen in CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP), Casalderrey-Solana, CE, Mateos, van der Schee arXiv:2011.08194 (JHEP)

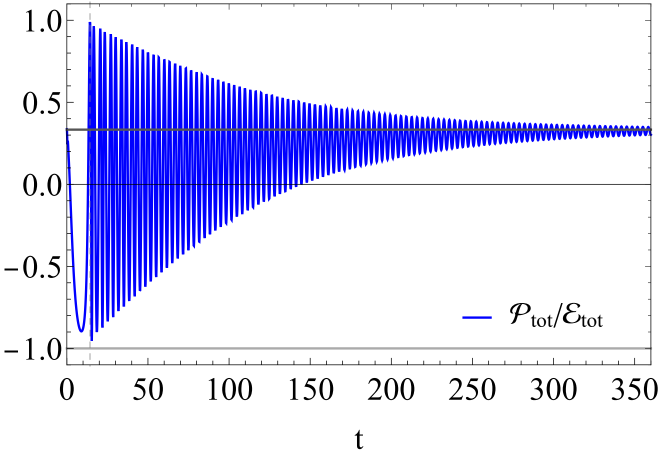
# Inflaton Pressure

- Inflaton acts initially like a pure cosmological constant $\mathcal{P}_{\text{inf}} = -\mathcal{E}_{\text{inf}}$.



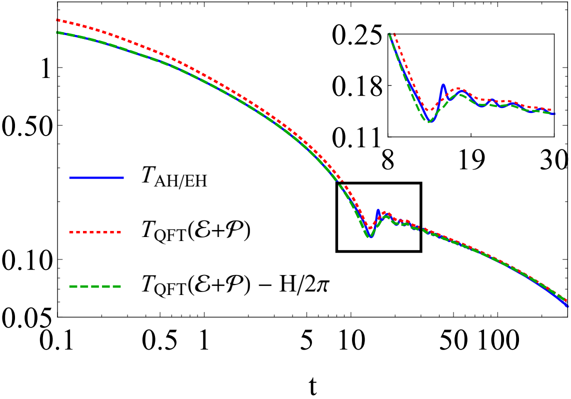
# Total Pressure

- The total pressure is initially dominated by the QFT, then by the inflaton and at late times again by the reheated QFT, which is then close to the conformal IR fixed point $\mathcal{P}_{\rm tot} \approx \mathcal{E}_{\rm tot}/3$.



# Temperature

- The QFT temperature can be computed from surface gravity of the bulk apparent horizon: $T_{\mathrm{AH}} = \kappa/2\pi$.
- Except for a short far-from-equilibrium period during reheating, the apparent and even horizon temperatures are numerically indistinguishable.
- Hydrodynamic approximation with EOS works well after subtracting de Sitter temperature of the cosmological horizon: $T_{\mathrm{dS}} = H/2\pi$.
  Consistent with exact CFT solution by Buchel, Heller, Noronha arXiv:1603.05344 (PRD)



# Summary

- The period between the end of inflation and big-bang nucleosynthesis is not well theoretically understood and observationaly constrained.
- Constructed a holographic framework that mimics the essential features of inflationary cosmology: inflationary phase, reheating and thermalization.
- Based on a combined action that assumes semi-classical boundary gravity and the standard holographic dictionary with mixed boundary conditions.
- Mathematica code available with all formulas (equations of motion, asymptotic series, holographic renormalization, . . . ) and numeric scheme.

https://sites.google.com/view/wilkevanderschee/public-codes

# Explicit Expressions for Holographic VEVs

$$ds^2 = L^2 \frac{\mathrm{d}\rho^2}{4\rho^2} + \bar{g}_{ij}(\rho, x)\mathrm{d}x^i \mathrm{d}x^j \,, \tag{34}$$

$$\begin{aligned} \bar{g}_{ij}(\rho, x) &= \frac{1}{\rho}\Big[\gamma_{ij}(x) + \rho\,\gamma_{(2)ij}(x) + \rho^2 \gamma_{(4)ij}(x) \\ &+ \rho^2 \log\rho\, h_{(4)ij}(x) + O(\rho^3)\Big]\,, \end{aligned} \tag{35}$$

$$\Phi(\rho, x) = \rho^{1/2}\Big[\Phi_{(0)}(x) + \rho\,\Phi_{(2)}(x) + \rho\,\log\rho\,\psi_{(2)}(x) + O(\rho^2)\Big]\,. \tag{36}$$

$$\begin{aligned} \langle T^{\mathrm{QFT}}_{ij} \rangle &= \frac{2}{\kappa_5}\Bigg\{\gamma_{(4)ij} + \frac{1}{8}\left[\mathrm{Tr}\gamma^2_{(2)} - (\mathrm{Tr}\gamma_{(2)})^2\right]\gamma_{ij} \\ &- \frac{1}{2}\gamma^2_{(2)} + \frac{1}{4}\gamma_{(2)ij}\mathrm{Tr}\gamma_{(2)} + \frac{1}{2}\partial_i\Phi_{(0)}\partial_j\Phi_{(0)} \\ &+ \left(\Phi_{(0)}\Phi_{(2)} - \frac{1}{2}\Phi_{(0)}\psi_{(2)} - \frac{1}{4}\partial_k\Phi_{(0)}\partial^k\Phi_{(0)}\right)\gamma_{ij} \\ &+ \alpha\left(\mathcal{T}^{\gamma}_{ij} + \mathcal{T}^{\phi}_{ij}\right) + \left(\frac{1}{18} + \beta\right)\Phi^4_{(0)}\gamma_{ij}\Bigg\}\,. \end{aligned} \tag{37}$$

$$\langle \mathcal{O} \rangle = \frac{2}{\kappa_5}\left[(1 - 4\alpha)\psi_{(2)} - 2\Phi_{(2)} - 4\beta\Phi^3_{(0)}\right]\,. \tag{38}$$

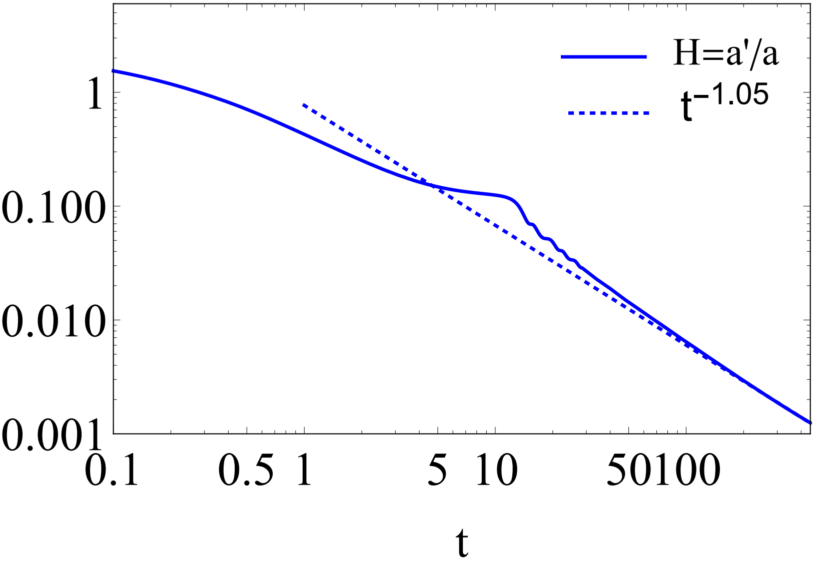
# Bulk Equations of Motion

$$\begin{aligned} \mathrm{d}s^2_{\mathrm{bulk}} &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2 \mathrm{d}\vec{x}^2\,, && (39) \\ \Phi &= \Phi(r,t)\,, && (40) \end{aligned}$$

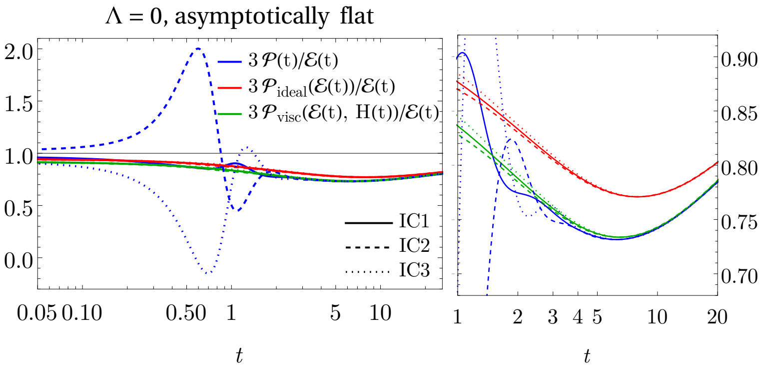
$$\begin{aligned} S'' &= -\frac{2}{3} S\left(\Phi'\right)^2\,, && (41) \\ \dot{S}' &= -\frac{2\dot{S}S'}{S} - \frac{2SV}{3}\,, && (42) \\ \dot{\Phi}' &= \frac{V'}{2} - \frac{3\dot{S}\Phi'}{2S} - \frac{3S'\dot{\Phi}}{2S}\,, && (43) \\ A'' &= \frac{12\dot{S}S'}{S^2} + \frac{4V}{3} - 4\dot{\Phi}\Phi'\,, && (44) \\ \ddot{S} &= \frac{\dot{S}A'}{2} - \frac{2S\dot{\Phi}^2}{3}\,, && (45) \end{aligned}$$

$$f' \equiv \partial_r f\,, \quad \dot{f} \equiv \partial_t f + \frac{1}{2}A\partial_r f\,. \qquad (46)$$

# Hubble Rate



# Hydrodynamization with frozen inflaton




CE, van der Schee, Mateos, Casalderrey-Solana arXiv:2109.10355 (JHEP)