Non-equilibrium steady state formation in 3+1 dimensions

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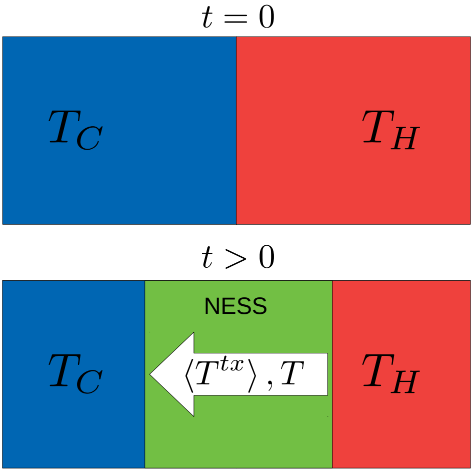
Based on

2103.10435 with Johanna Erdmenger and Wilke van der Schee

Non-equilibrium steady states

- NESS: time independent thermal properties, but non-vanishing fluxes.



CFT_2

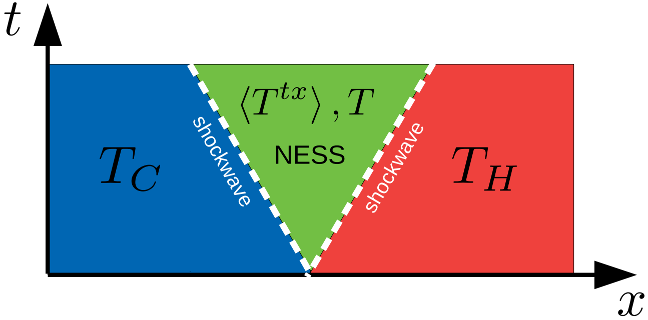
- Universal formula for heat current and temperature

$$\langle T^{tx} \rangle = \frac{\pi c}{12}\left(T_C^2 - T_H^2\right)\,, \quad T = \sqrt{T_C T_H}\,, \quad v_{H/C} = \pm 1\,.$$

[Bernard, Doyon (1202.0239)]

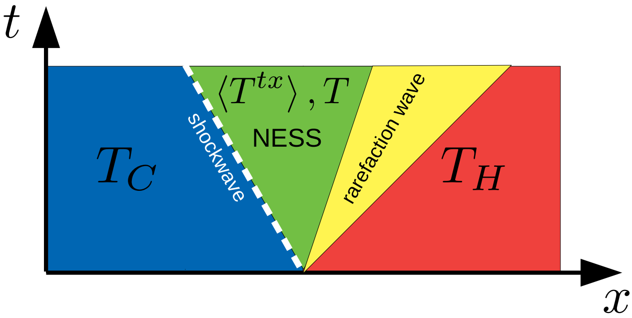
- Recently generalised to $T\bar{T}$-deformed CFT_2.
[Medenjak, Policastro, Yoshimura (2011.05827)]

- $\mathrm{AdS}_3/\mathrm{CFT}_2$: gravity dual of NESS is boosted BTZ black brane.
[Bhaseen, Doyon, Lucas, Schalm (1311.3655)]

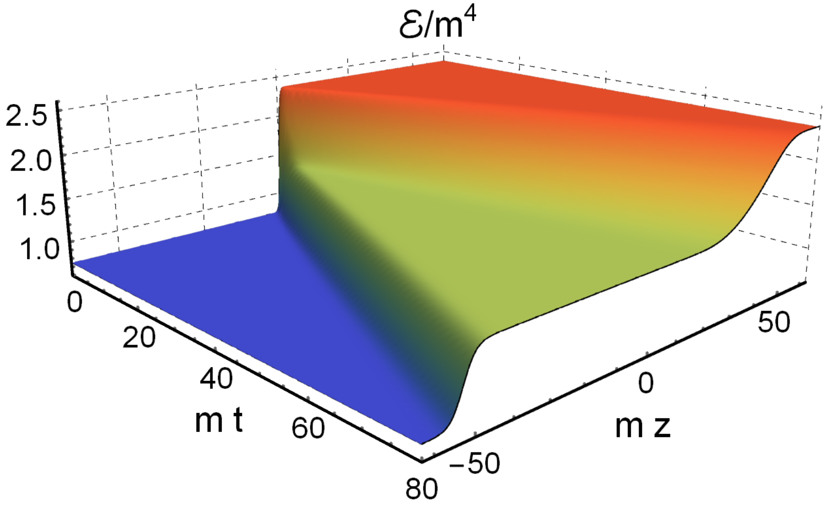


$D > 2$

- No closed solutions available, but insights from hydro and holography. [Spillane, Herzog (1512.09071); Amado, Yarom (1501.01627); Pourhasan (1509.01162); Herzog, Spillane, Yarom (1605.01404); Fernandez, Rajagopal, Thorlacius (1909.06377)]
- Shockwave moving towards hot side violates local version of the 2nd law of thermodynamics. Can be cured by replacing shock with rarefaction wave. [Lucas, Schalm, Doyon, Bhaseen (1512.09037); Spillane, Herzog (1512.09071); Glorioso, Liu (1612.07705)]
- Boosted black branes are the only regular solutions with homogeneous stress tensor $\Rightarrow$ NESS region is described by boosted density matrix. [Bhaseen, Doyon, Lucas, Schalm (1311.3655)]



AdS_5/CFT_4



Riemann problem

Ideal hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu J^\mu = 0\,, \quad T^{\mu\nu} = (\mathcal{E} + \mathcal{P}) u^\mu u^\nu + \mathcal{P}\eta^{\mu\nu}\,, \quad J^\mu = n u^\mu\,.$$

Equation of state of a conformally invariant fluid

$$\mathcal{P}(\mathcal{E}) = c_s^2 \mathcal{E}\,, \quad c_s^2 = \frac{1}{d}\,.$$

Initial value problem with discontinuous ICs for energy and charge density

$$\mathcal{E}(0,x) = \begin{cases} \mathcal{E}_C & \forall x < 0 \\ \mathcal{E}_H & \forall x > 0 \end{cases}, \quad n(0,x) = \begin{cases} n_C & \forall x < 0 \\ n_H & \forall x > 0 \end{cases}.$$

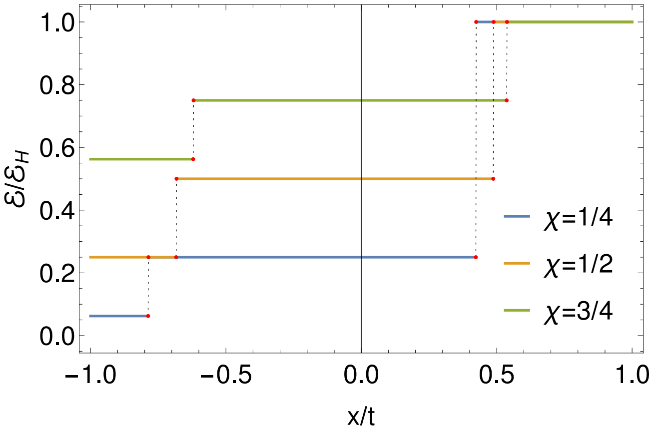
Weak solutions including discontinuities only need to satisfy integrated EOMs

$$\int_0^\infty \mathrm{d}t \int_{-\infty}^\infty \mathrm{d}x\, \phi \partial_\mu T^{\mu\nu} = 0\,, \quad \int_0^\infty \mathrm{d}t \int_{-\infty}^\infty \mathrm{d}x\, \phi \partial_\mu J^\mu = 0\,.$$

Double-shock solution

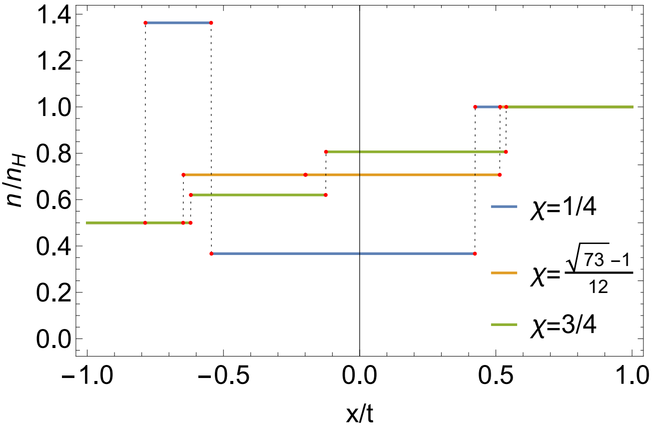
$$v_C = -c_s\sqrt{\frac{1+c_s^2\chi}{c_s^2+\chi}}\,, \quad v_S = -\frac{c_s(1-\chi)}{\sqrt{(1+c_s^2\chi)(c_s^2+\chi)}}\,, \quad v_H = c_s\sqrt{\frac{c_s^2+\chi}{1+c_s^2\chi}}\,,$$

$$\mathcal{E}_S = \sqrt{\mathcal{E}_C\mathcal{E}_H}\,, \quad \chi = \sqrt{\mathcal{E}_C/\mathcal{E}_H}$$



Contact discontinuity

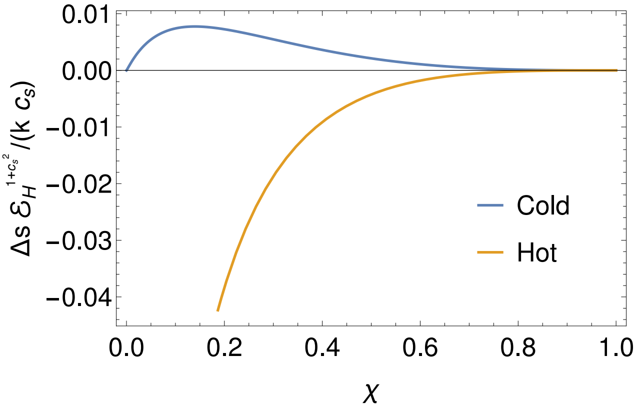
$$n_1 = n_C\sqrt{\frac{1+c_s^2\chi}{\chi(c_s^2+\chi)}}\,, \quad n_2 = n_H\sqrt{\frac{\chi(c_s^2+\chi)}{1+c_s^2\chi}}$$



Entropy condition

- Shock wave moving into hot region violates the local entropy condition

$$\partial_\mu s^\mu \geqslant 0\,, \quad s^\mu = \frac{\mathcal{E}+\mathcal{P}}{T}u^\mu = k\mathcal{E}^{\frac{1}{1+c_s^2}}u^\mu\,.$$

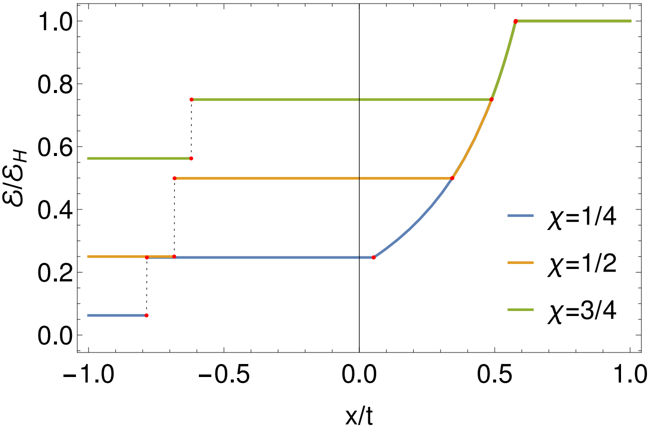


$$\lim_{\chi\to 0} \Delta s_H = -k\, c_s^2\, (\mathcal{E}_H)^{\frac{1}{1+c_s^2}}\,, \quad \chi_2^* \approx 0.1301\,, \chi_3^* \approx 0.1397\,, \chi_4^* \approx 0.1428\,, \ldots$$

Rarefaction wave

- A rarefaction wave saturates the entropy condition by construction

$$\partial_\mu s^\mu = 0\,.$$



Holographic model

- 5D Einstein-Maxwell gravity

$$S = \frac{1}{16\pi G_N} \int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{e^2L^2}{4}F_{MN}F^{MN}\right) + S_{\mathrm{GHY}} + S_{\mathrm{ct}}\,.$$

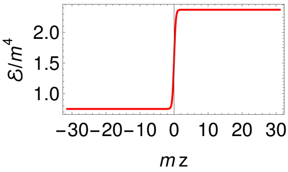
- Inhomogeneous AdS_5 black branes

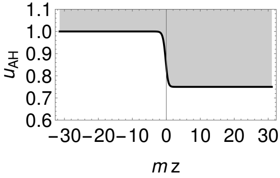
$$ds^2 = -C\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + 2G\mathrm{d}z\mathrm{d}t + S^2\left(e^B\mathrm{d}x_\perp^2 + e^{-2B}\mathrm{d}z^2\right)\,, \quad A_M = A_t\mathrm{d}t + A_z\mathrm{d}z\,.$$

- Inhomogenous states in $\mathcal{N} = 4$ SYM

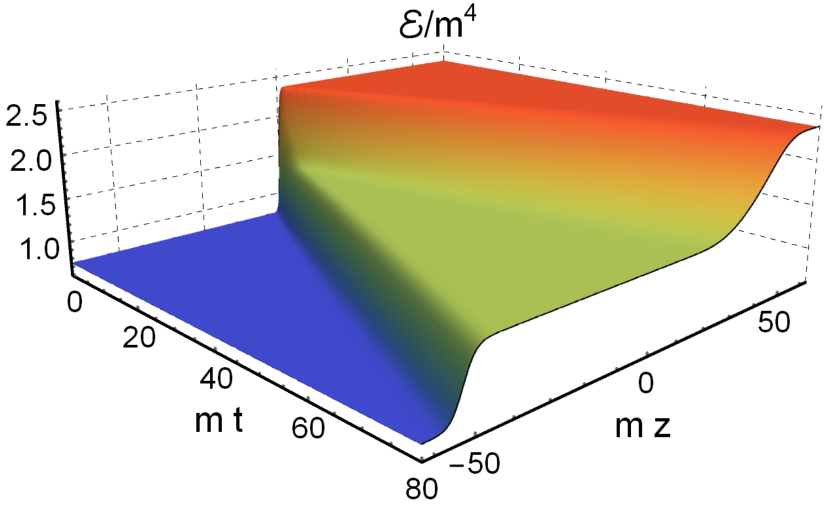
$$\langle T^{\mu\nu}\rangle = \frac{1}{4\pi G_N}\begin{pmatrix} \mathcal{E} & \mathcal{S} & 0 & 0 \\ \mathcal{S} & \mathcal{P}_\parallel & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ 0 & 0 & 0 & \mathcal{P}_\perp \end{pmatrix}, \quad \langle J^\mu\rangle = \frac{e^2L^4}{4\pi G_N}\begin{pmatrix} \overline{\rho} \\ \overline{\sigma} \\ 0 \\ 0 \end{pmatrix}.$$

- IC: Smoothed step function for energy density $\Rightarrow$ inhomogeneous horizon.

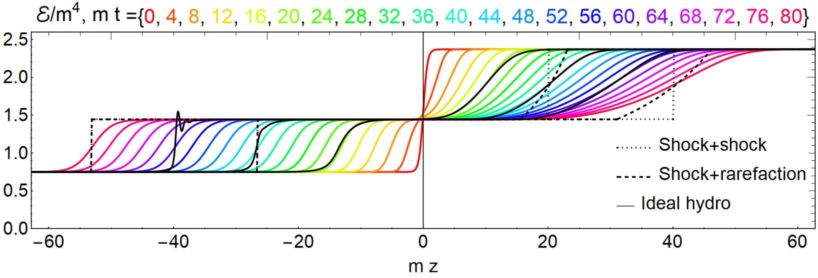




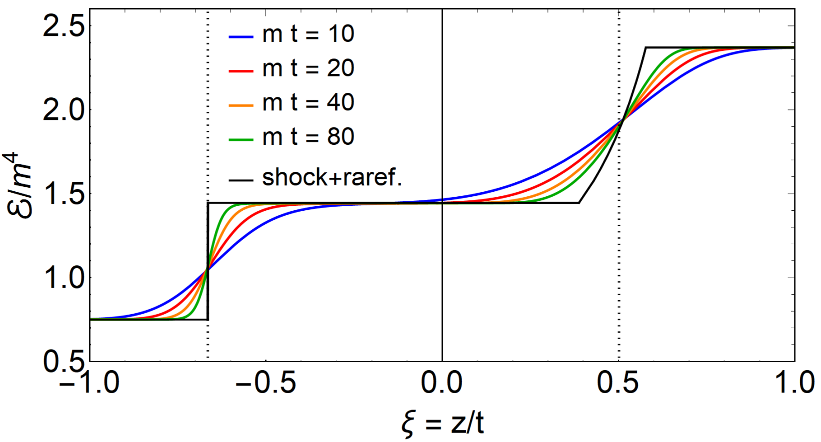
Energy density



Energy density



Energy density

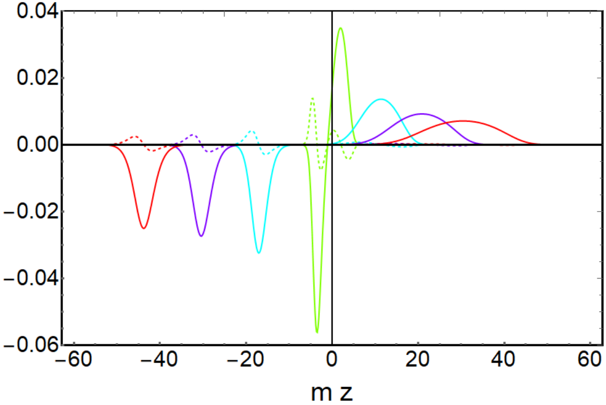


Deviation from hydrodynamics

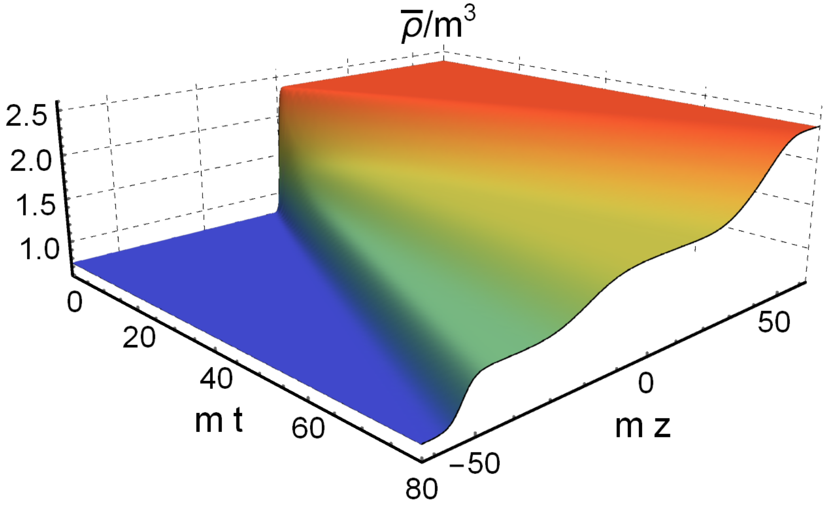
Check constituent relations for $\mathcal{N} = 4\,\mathrm{SYM}$: $\eta/s = 1/4\pi\,, \zeta = 0\,, \mathcal{P} = \frac{1}{3}\mathcal{E}$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\partial_\rho u^\rho \Delta^{\mu\nu} + \ldots$$

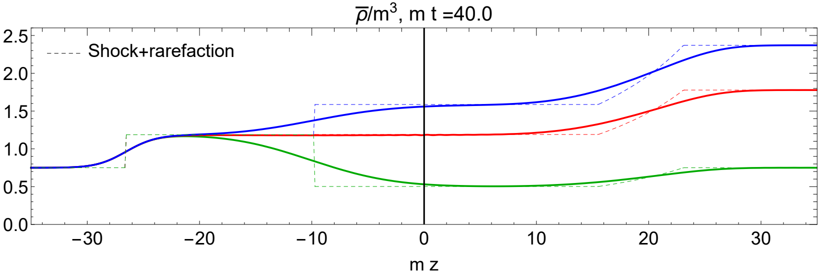
$\frac{\mathcal{P}_T - \mathcal{P}_{T,hyd}}{\mathcal{P}_T}$ (solid ideal, dashed viscous), m t ={5, 25, 45, 65}



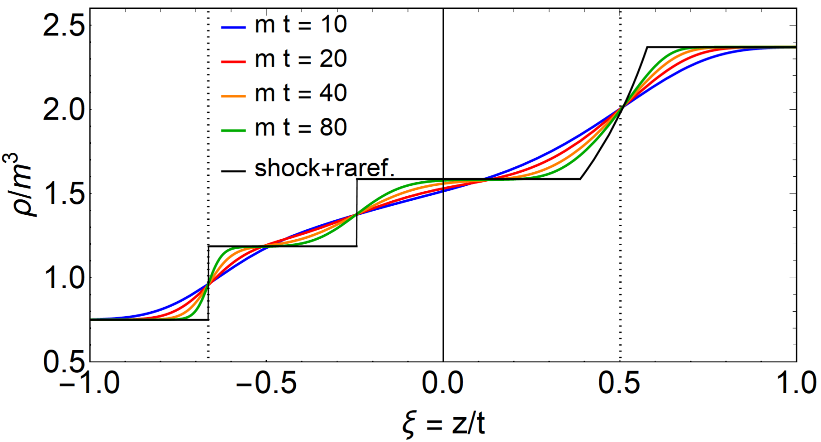
Charge density



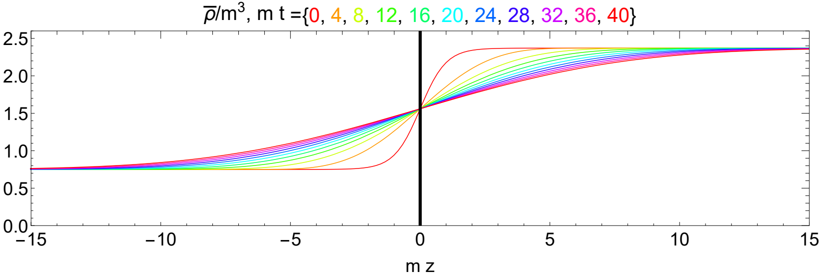
Charge density



Charge density



Diffusion of charge

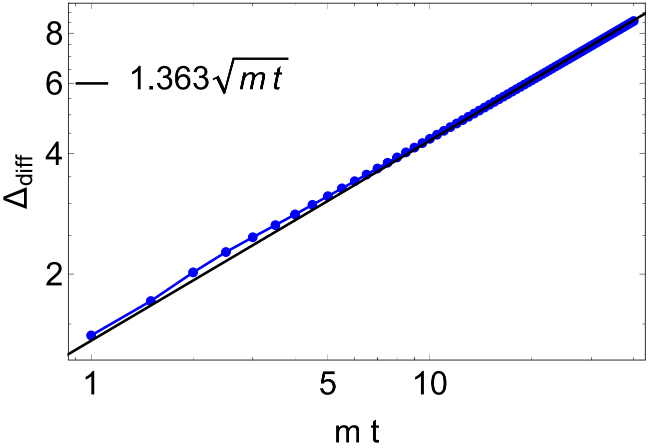


Diffusion of charge

- Expected result for viscous diffusion: $\Delta_{\mathrm{diff}}(t) \approx \sqrt{Dt}$.

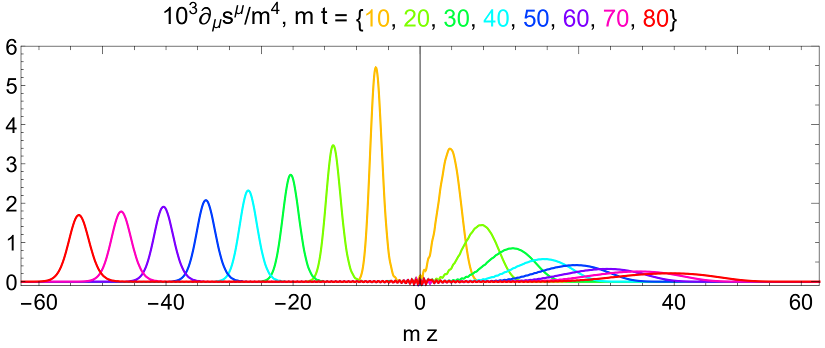
  [Lucas, Schalm, Doyon, Bhaseen (1512.09037)]

- Estimate width of the shock $\Delta_{\mathrm{diff}}(t)$ by distance between points where charge density is 25% and 75% of its maximum value.



Entropy production

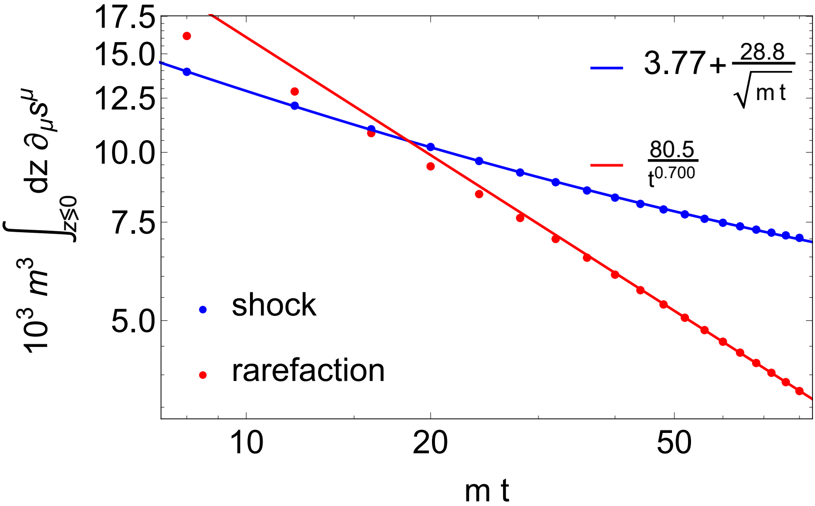
- Dual gravity description automatically satisfies entropy condition: $\partial_\mu s^\mu \geqslant 0$ translates to Raychaudhuri equation at the horizon. [Eling, Oz (1103.1657)]



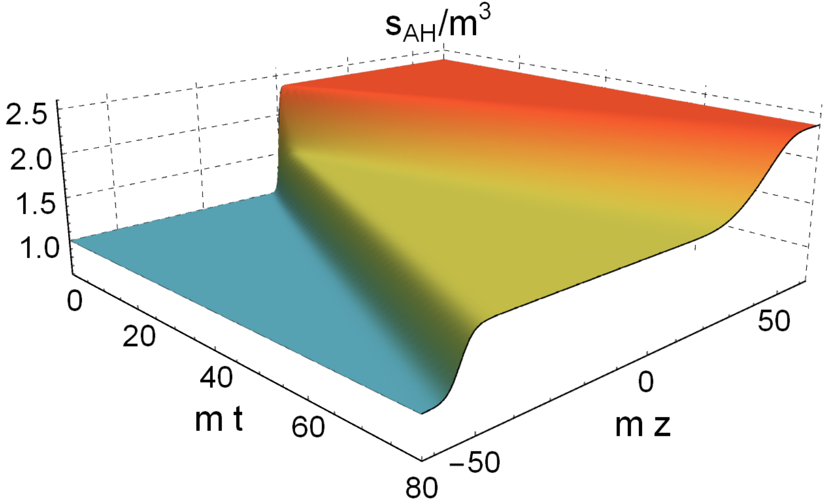
Entropy production

Rarefaction wave: $\lim_{t\to\infty} \frac{80.5}{t^{0.700}} = 0$ consistent with $\partial_\mu s^\mu = 0$.

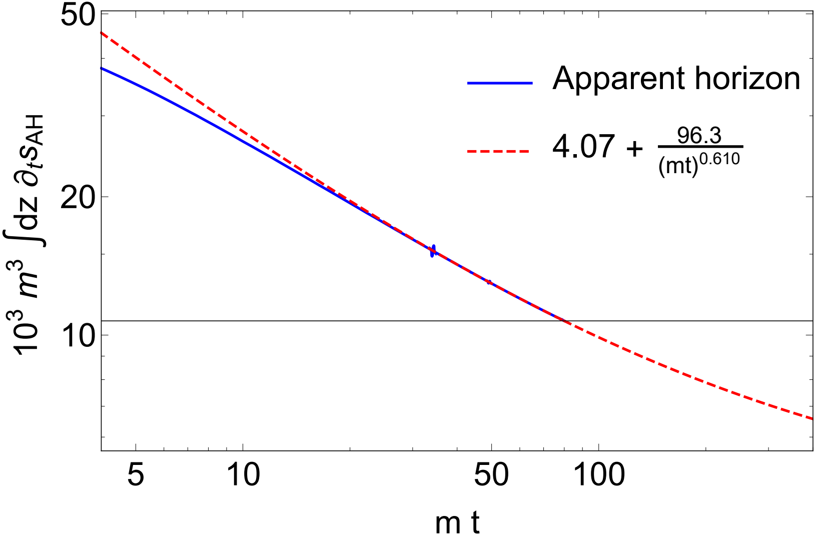
Shockwave: $\int_{z<0} dz\, \partial_\mu s^\mu = \frac{\pi}{\sqrt{3} m^3}\left(\chi^{-1/4} - \sqrt{\frac{\chi(3+\chi)}{1+3\chi}}\right) \approx 0.00610.$



Apparent horizon entropy



Apparent horizon entropy



Entanglement entropy

- Divide system into two parts $(A, B = \bar{A})$
- Assume that the Hilbert space factorizes

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

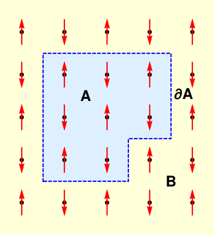
- Compute reduced density matrix by tracing over $\mathcal{H}_B$

$$\rho_A = \mathrm{Tr}_B\, \rho$$

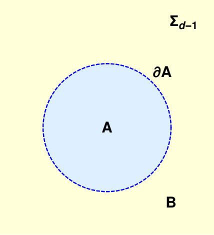
- Entanglement entropy is defined as the von Neumann entropy of ρ_A

$$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A$$

Lattice Theory



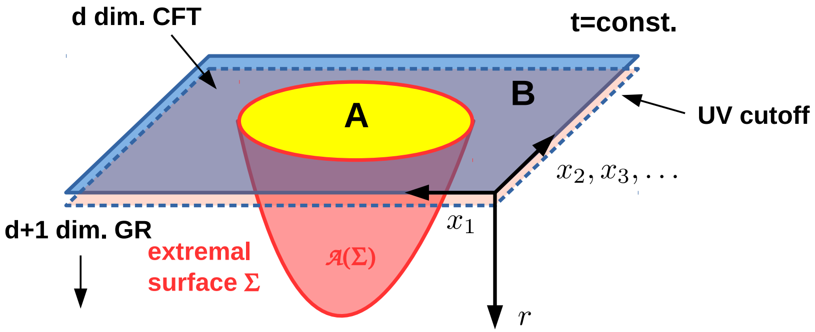
Quantum Field Theory



Holographic entanglement entropy

$$S_A = \frac{\mathcal{A}}{4G_N}$$

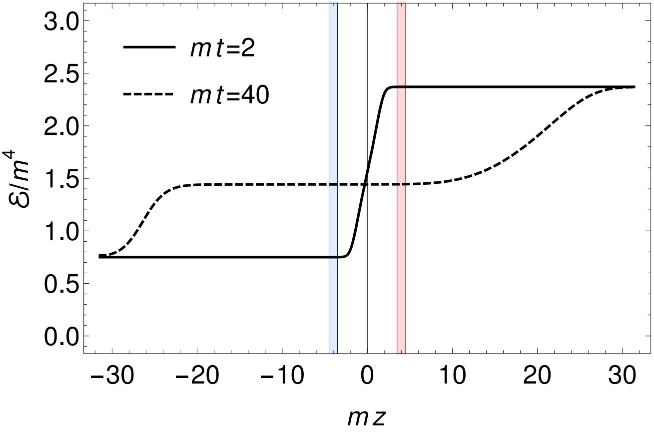
[Ryu, Takayanagi (hep-th/0603001); Hubeny, Rangamani, Takayanagi (0705.0016)]



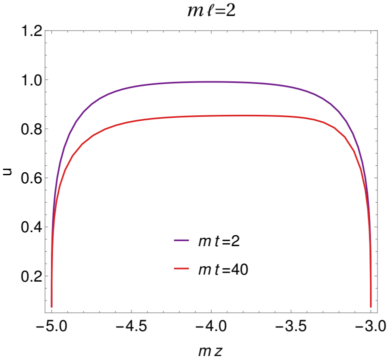
Infinite stripe regions

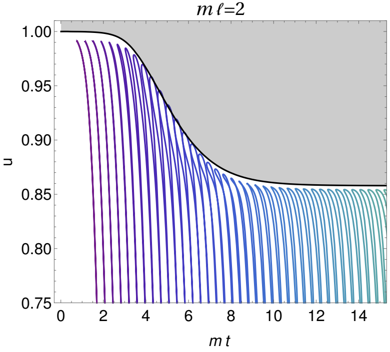
$$\mathcal{A}[X] = \ell_\perp^2 \int \mathrm{d}\sigma \sqrt{\bar{g}_{\alpha\beta}(U(\sigma), T(\sigma), Z(\sigma)) \frac{\mathrm{d}X^\alpha}{\mathrm{d}\sigma} \frac{\mathrm{d}X^\beta}{\mathrm{d}\sigma}} \quad \text{s.t.} \quad X^\alpha(0) = \{0, t_0, \pm\ell/2\} \,,$$

$$\frac{\mathrm{d}^2 X^\alpha}{\mathrm{d}\sigma^2} + \Gamma^\alpha_{\beta\gamma} \frac{\mathrm{d}X^\beta}{\mathrm{d}\sigma} \frac{\mathrm{d}X^\gamma}{\mathrm{d}\sigma} = J \frac{\mathrm{d}X^\alpha}{\mathrm{d}\sigma} \,, \quad S_{\mathrm{ren}} = \frac{\mathcal{A}^{\mathrm{cut}} - \mathcal{A}_0^{\mathrm{cut}}}{4 G_{\mathrm{N}} \ell_\perp^2} \,.$$



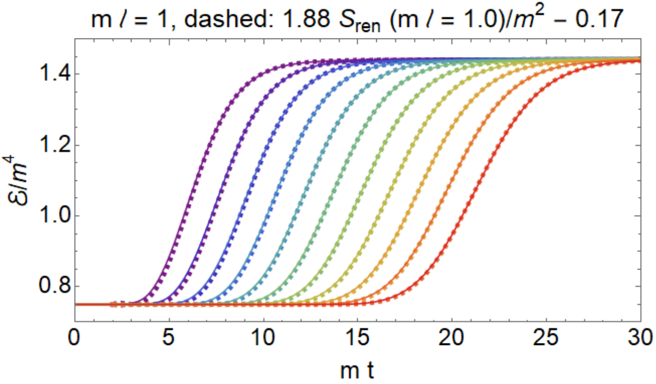
Extremal surfaces



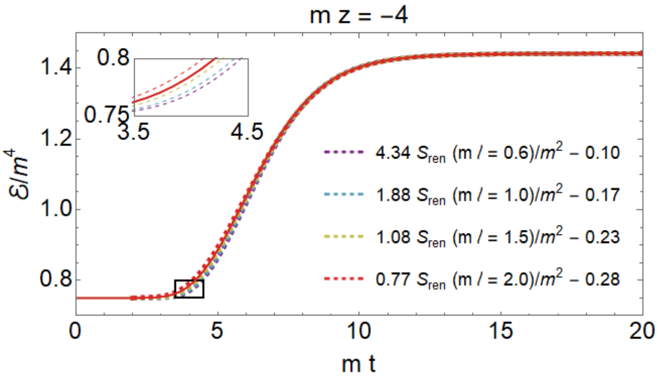


Entanglement entropy for shock

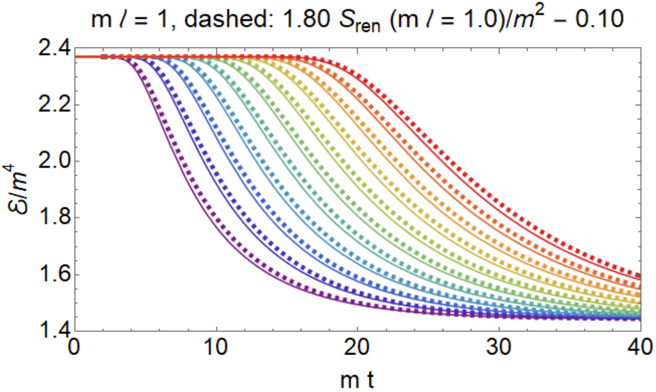
$$aS_{\mathrm{ren}} + b \approx \mathcal{E}$$



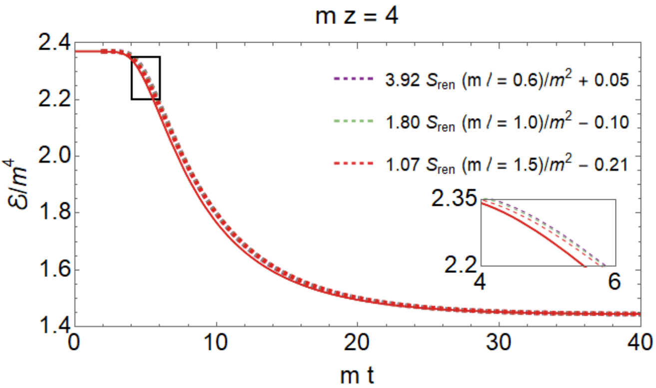
Entanglement entropy for shock



Entanglement entropy for rarefaction wave



Entanglement entropy for rarefaction wave



Summary

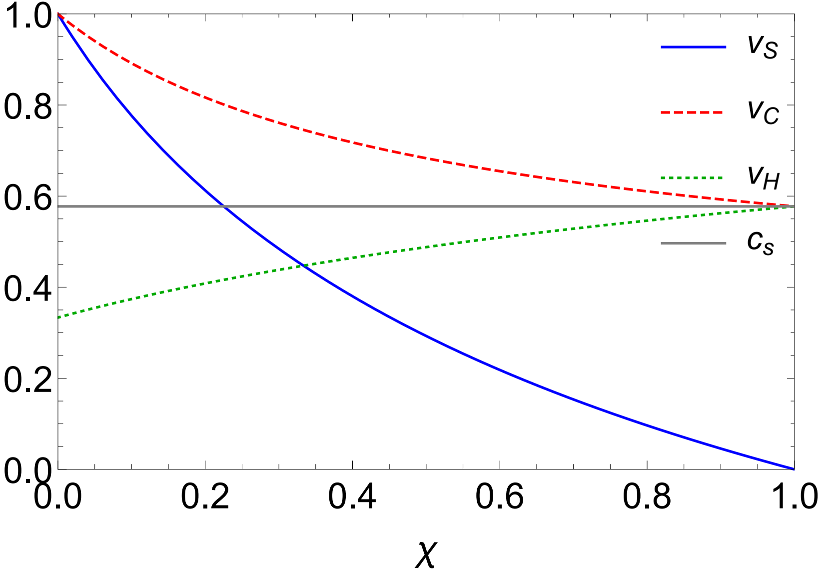
- Shock+rarefaction wave solution of the Riemann problem provides excellent approximation to the properties of NESSs in $\mathcal{N} = 4$ SYM.
- Only small ($\lessapprox 1\%$) deviations from constituent relations of viscous hydrodynamics at the locations of shocks.
- Charge density forms diffusing transition region inside NESS, no evidence for sharp contact discontinuity.
- HEE follows closely the energy density, except for a short delay.

Open problems

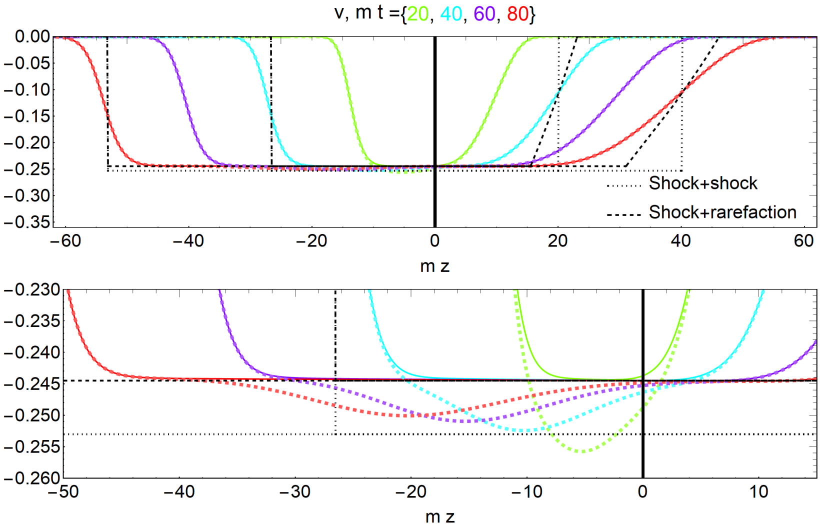
- Backreaction of the gauge field $\Rightarrow \mathcal{P} = \mathcal{P}(\mathcal{E}, \rho)$, what happens to NESS when diffusing charge backreacts on energy?
- Transverse flow $\Rightarrow$ contact discontinuity in energy density.
  [Mach (1104.3751)]
- Scalar potential for phase transitions and conformal symmetry breaking.
  [Bea, Casalderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia (2104.05708)]
- Probe deeper into the far-from-equilibrium regime $\chi \ll 1$.

Backup

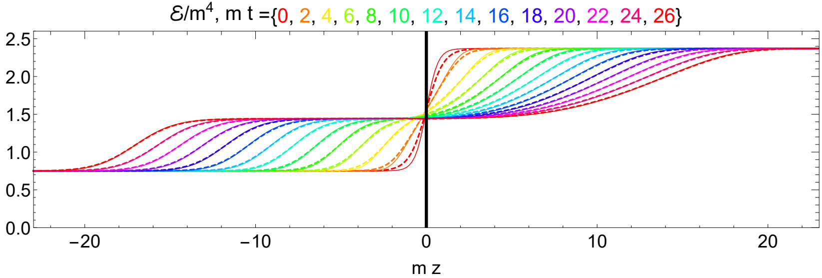
Shock Velocities



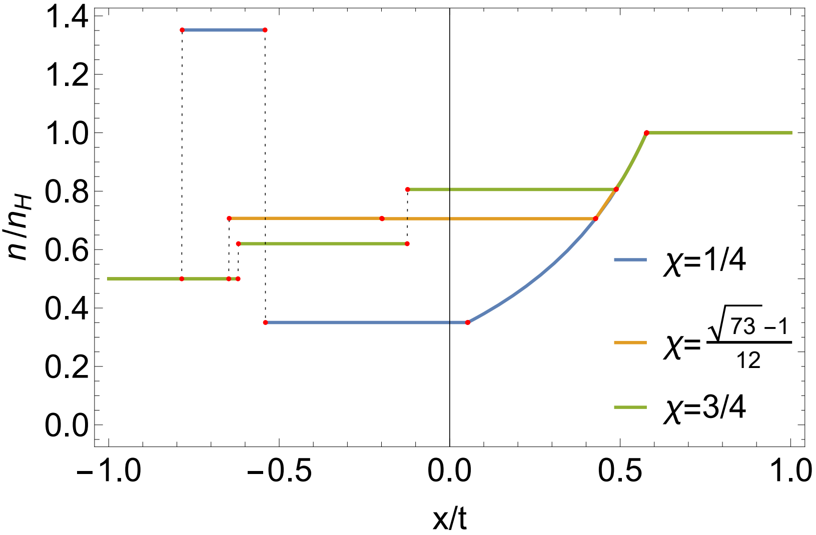
Velocities



Independence of initial conditions



Charge density



Comparison to Hydro II