On the Sound Speed in Neutron Stars

Christian Ecker

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Based on
2203.14974 with Sinan Altiparmak and Luciano Rezzolla

Motivation

- Adiabatic speed of sound[1] provides a measure for the stiffness of matter

$$c_s^2 = \left(\frac{\partial p}{\partial e}\right)_s \,. \tag{1}$$

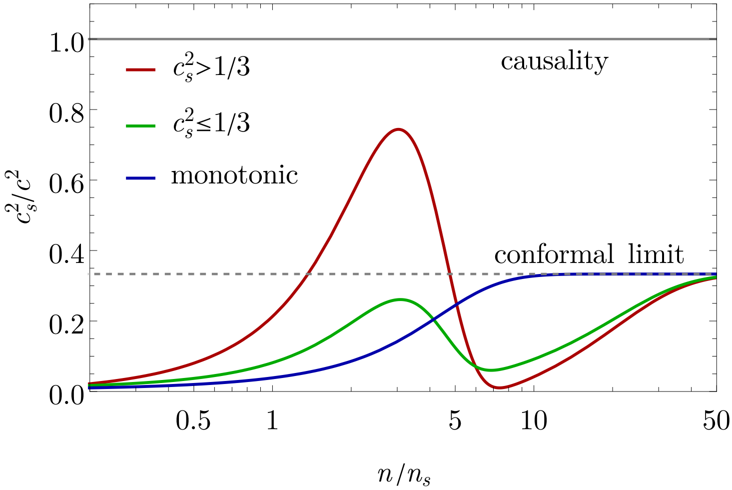
- Relevant for neutron star properties: stiff (soft) matter has large (small) sound speed and results in large (small) maximal mass and radius.
- Constrained by causality and thermodynamic stability: $c_s^2 \in [0, 1]$.
- "Slope" of the Equation of State (EOS) only known in certain limits:

$$c_s^2(n) = \begin{cases} \ll 1 & \forall\, n \ll n_s\,, \\ ? & \forall\, n \gtrsim n_s\,, \quad n_s = 0.16\,\mathrm{fm}^{-3}\,, \\ \to 1/3 & \forall\, n \gg n_s\,. \end{cases} \tag{2}$$

- Difficult open problem: What is $c_s^2(n)$ in QCD at neutron star densities?

[1] We use units where we set the speed of light $c = 1$.

Several different possibilities



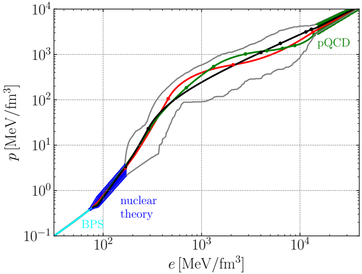
Making progress without solving QCD

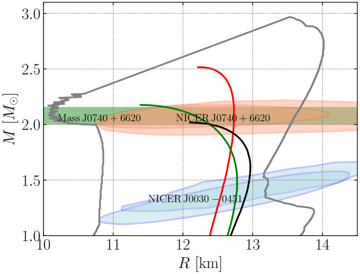
- Use nuclear theory at low and perturbative QCD results at high densities.
  [Baym, Pethick, Sutherland (1971), Hebeler, Lattimer, Pethick, Schwenk arXiv:1303.4662, Fraga, Kurkela, Vuorinen arXiv:1311.5154]
- Randomly generate several linear speed of sound segments in between.

$$c_s^2(\mu) = \frac{(\mu_{i+1}-\mu)c_{s,i}^2 + (\mu-\mu_i)c_{s,i+1}^2}{\mu_{i+1}-\mu_i}\,, \; n(\mu) = n_1\, e^{\int_{\mu_1}^{\mu} \frac{\mathrm{d}\mu'}{\mu' c_s^2(\mu')}}\,, \; p(\mu) = p_1 + \int_{\mu_1}^{\mu} \mathrm{d}\mu'\, n(\mu')\,. \tag{3}$$

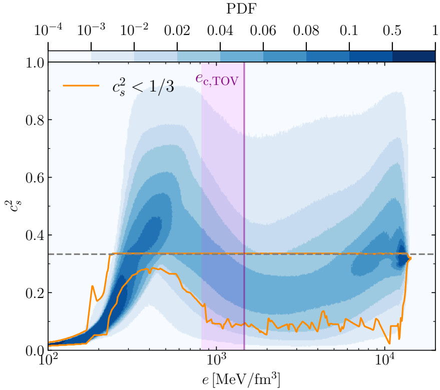
[Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:1903.09121]

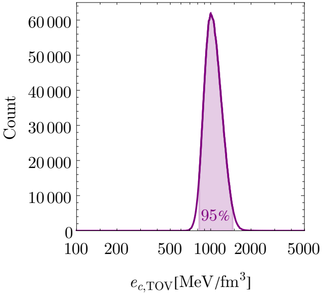
- Solve TOV and keep only models that satisfy observational constraints.
- Repeat several million times and study the distribution of viable solutions.





Sound speed distribution of more than 10^6 models



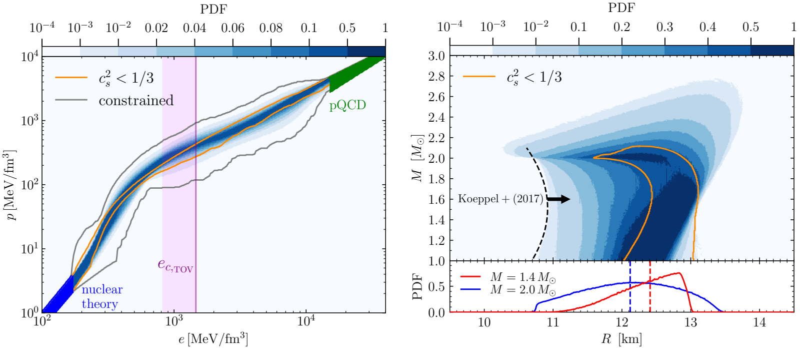


- Local maximum $c_s^2 > 1/3$ below maximal central densities of TOV stars

$$e_{c,\mathrm{TOV}} = 1064^{+399}_{-244}\,\mathrm{MeV/fm^3}\;(95\%\,\text{confidence})\,. \qquad (4)$$

- $c_s^2 \leqslant 1/3$ at all densities possible, but only a small fraction (0.03%).
- Lower bound $c_s^2 \gtrsim 0.2$ of sub-conformal models around $e \approx 400\,\mathrm{MeV/fm^3}$.

EOS and mass-radius distribution



- Outer EOS contour like Annala+, but much narrower distribution.

[Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:2105.05132]

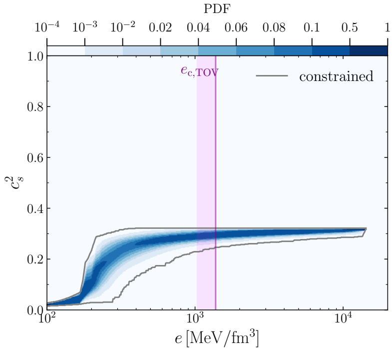
- Upper bound of generic models: $M_{\mathrm{TOV}} \lesssim 3M_\odot$.
- Upper bound of sub-conformal models: $M_{\mathrm{TOV}} < 2.1M_\odot$.
- Radius estimates (+ error bars) for typical stars:

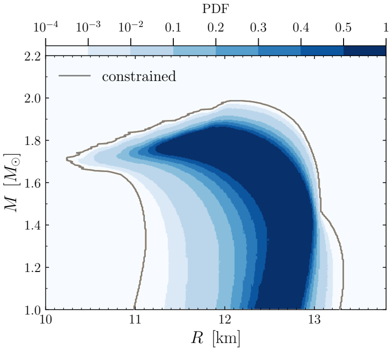
$$R_{1.4} = 12.42^{+0.52}_{-0.99}\,\mathrm{km}\,,\ R_{2.0} = 12.11^{+1.11}_{-1.23}\,\mathrm{km}\,(95\%\,\mathrm{confidence})\,. \qquad (5)$$

- Remarkable agreement with $R_{min}(M)$ from threshold mass calculation.

[Koeppel, Rezzolla arXiv:1901.09977]

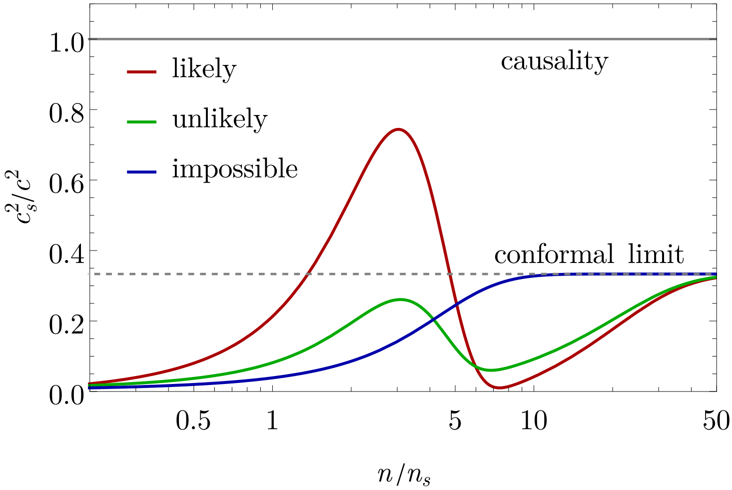
Monotonic sound speed





- Separate calculation for more than 10^7 models.
- Drop condition on the lower bound on maximal mass.
- Monotonic models can only reach $M_{TOV} < 2\,M_\odot$.

Several Possibilities



Binary tidal deformability and chirp mass I

- Chirp mass can be extracted accurately from inspiral GW strain signal

$$\mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = \left(\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right)^{3/5} . \tag{6}$$

- Binary tidal deformability encodes EOS properties via Λ_i

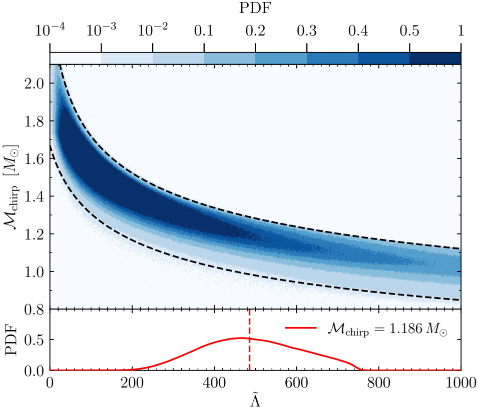
$$\tilde{\Lambda} = \frac{16}{13} \frac{(12 M_2 + M_1)\, M_1^4 \Lambda_1 + (12 M_1 + M_2)\, M_2^4 \Lambda_2}{(M_1 + M_2)^5} . \tag{7}$$

- GW170817 only "good" event so far:

$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002} M_\odot , \quad q = M_2/M_1 > 0.7 , \quad \tilde{\Lambda} \lesssim 720 . \tag{8}$$

[LIGO/Virgo: arXiv:1805.11579]

Binary tidal deformability and chirp mass II



- More than 10^8 binary configurations with $M_{1,2} \in [0.5 M_\odot, M_{\rm TOV}]$.
- Simple fitting formula for the minimum and maximum of $\tilde{\Lambda}(\mathcal{M}_{chirp})$

$$\tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}_{chirp}^{c}\,, \tag{9}$$

$$a = -50(-20),\ b = 500(1800),\ c = -4.5(-5.0)\,.$$

Summary

- Studied sound speed distribution of several million viable EOS models.
- Sound speed is non-monotonic and probably $c_s^2 > 1/3$ in stellar interior.
- $c_s^2 \leqslant 1/3$ possible but only 0.03% of more than 10^6 viable EOSs.
- Simple fitting formulas for upper and lower bound on $\tilde{\Lambda}(\mathcal{M}_{\mathrm{chirp}})$.

[Altiparmak, CE, Rezzolla arXiv:2203.14974]

In progress:

- Study models with first order phase transition. Can we constrain the onset and strength of first order phase transitions with observational data?
- Bayesian analysis for observational constraints and EOS parameters.

[CE, Jiang, Rezzolla]

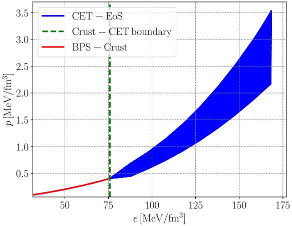
Building blocks of the EOSs

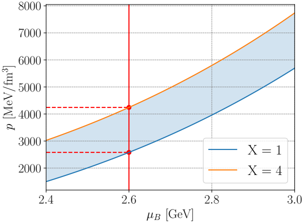
- Lowest densities ($0 - 0.5\, n_s$): Baym-Pethick-Sutherland (BPS) EoS
  [Baym, Pethick, Sutherland (1971)]
- Low densities ($0.5 - 1.2\, n_s$): $p(n) = K\, n^{\Gamma}\,, \quad \Gamma \in [1.77, 3.23]$
  [Hebeler, Lattimer, Pethick, Schwenk 1303.4662 (ApJ)]
- High densities ($1.2 - 40\, n_s$): speed of sound parametrization
  [Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:1903.09121]
- Highest densities ($\gtrsim 40\, n_s$): perturbative QCD results

$$p(\mu) = \frac{3}{4\pi^2}\left(\frac{\mu}{3}\right)^4 \left(c_1 - \frac{d_1 X^{-\nu_1}}{\mu/GeV - d_2 X^{-\nu_2}}\right)\,, \quad X \in [1,4]\,, \tag{10}$$

$$c_1 = 0.9008 \quad d_1 = 0.5034 \quad d_2 = 1.452 \quad \nu_1 = 0.3553 \quad \nu_2 = 0.9101\,.$$

[Kurkela, Romatschke, Vuorinen arXiv:0912.1856; Fraga, Kurkela, Vuorinen arXiv:1311.5154]





Observational constraints

- NICER: Recent radius measurement of PSR J0740+6620:

$$M_{\rm TOV} > 2.08^{+0.07}_{-0.07}(2.072^{+0.067}_{-0.066})M_\odot \ , \ R = 13.7^{+2.6}_{-1.5}(12.39^{+1.3}_{-0.98})\text{km}.$$

[Miller et al. arXiv:2105.06979, (Riley et al. arXiv:2105.06980)]

- LIGO/Virgo: constraints on tidal deformability from GW170817:

$$\tilde{\Lambda} < 720 \text{ for } \mathcal{M}_{chirp} = 1.186 M_\odot \text{ and } q > 0.73$$

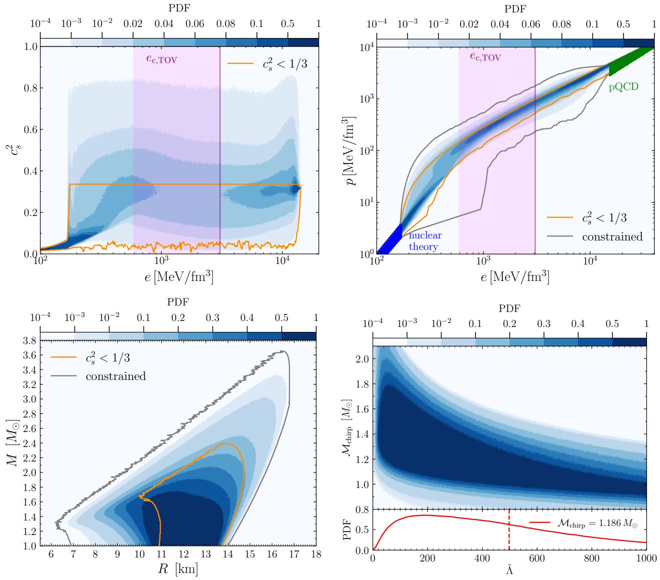
[LIGO/Virgo: arXiv:1710.05832, arXiv:1805.11579, arXiv:1805.11581]

- Radius measurement of PSR J0030+00451:

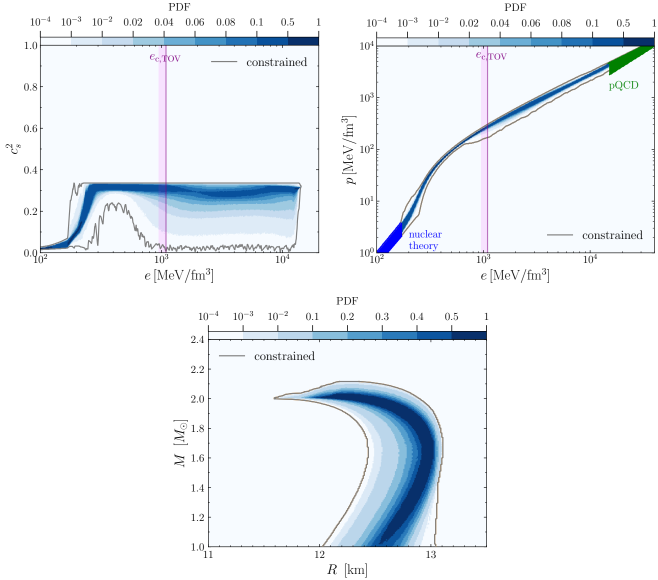
$$M = 1.34^{+0.15}_{-0.16}(1.44^{+0.15}_{-0.14})M_\odot \ , \ R = 12.71^{+1.14}_{-1.19}(13.02^{+1.24}_{-1.06})\text{km}$$

[Riley et al. arXiv:1912.05702, (Miller et al. arXiv:1912.05705)]

Without observational constraints



Sub-conformal models



Different number of segments

- Sound speed and MR when using 3,4,5 and 7 segments.
- Features at neutron star densities are robust.