Exploring the Phase Diagram of V-QCD with Neutron Star Mergers

Christian Ecker

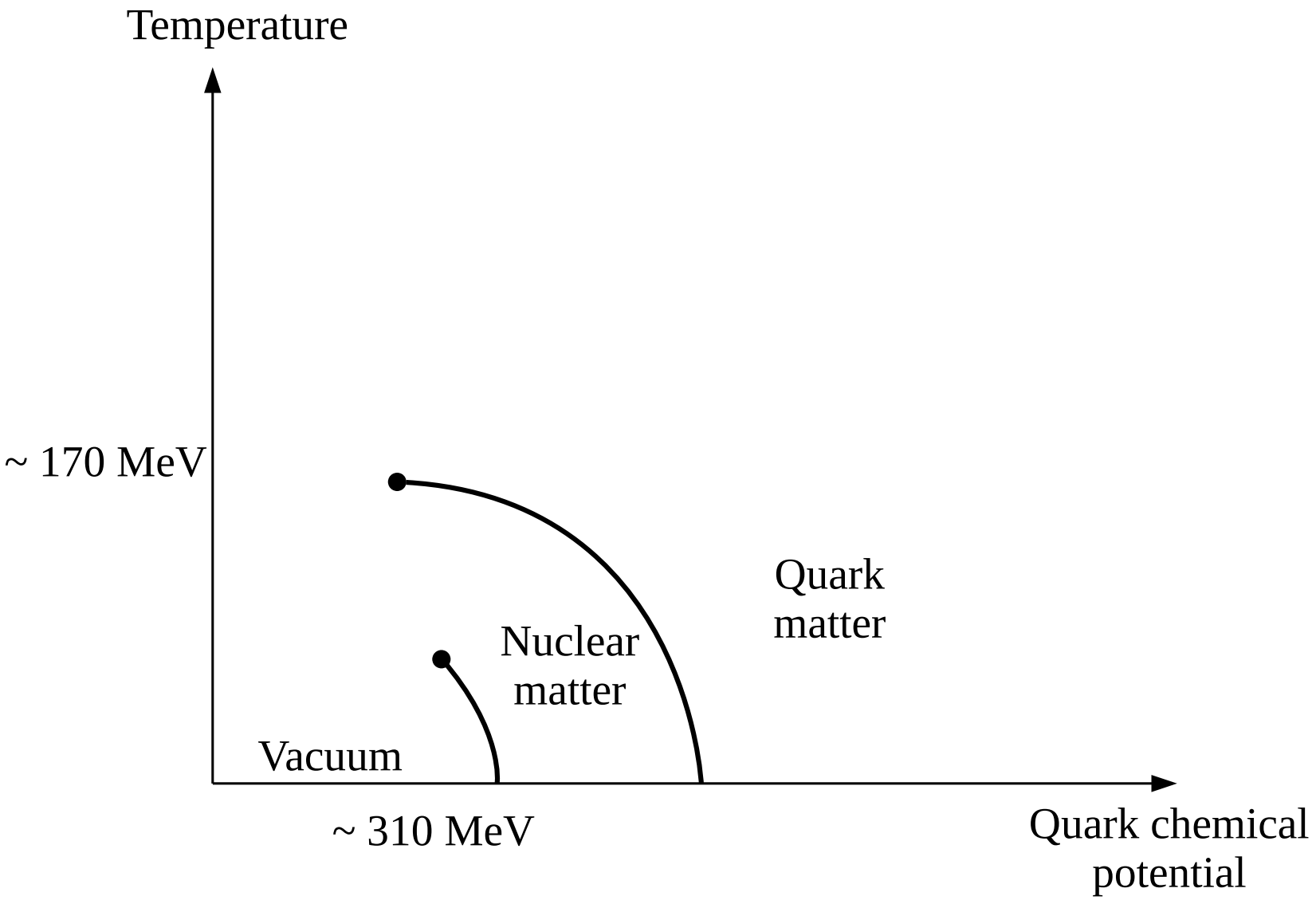
1 August 2022

Based on 2112.12157 and 2205.05691 with

Tuna Demircik, Matti Järvinen, Luciano Rezzolla, Samuel Tootle and Konrad Topolski

See also talk by Matti Järvinen on August 5 2:25 PM (Track G)!

What we (don't) known about QCD



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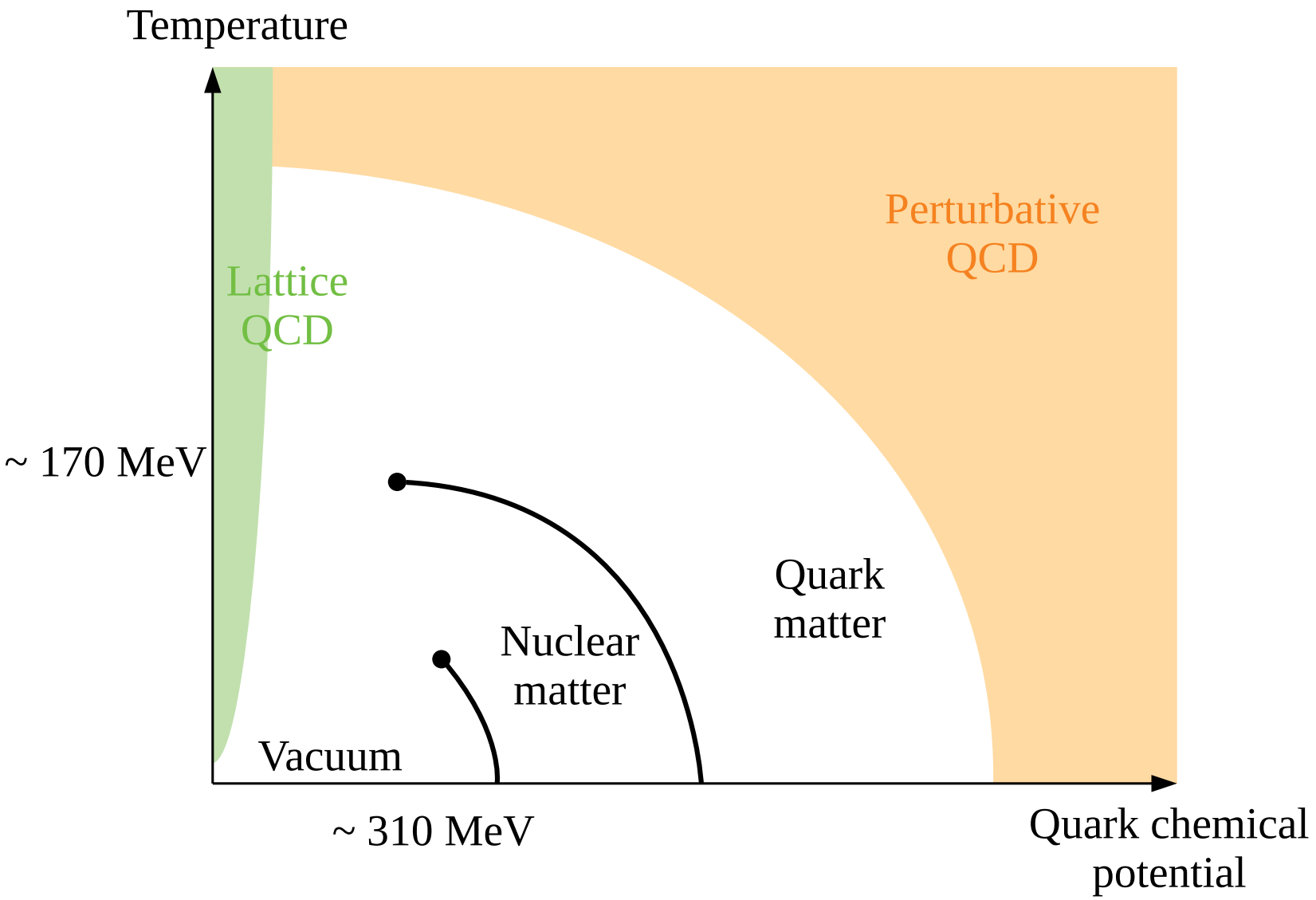

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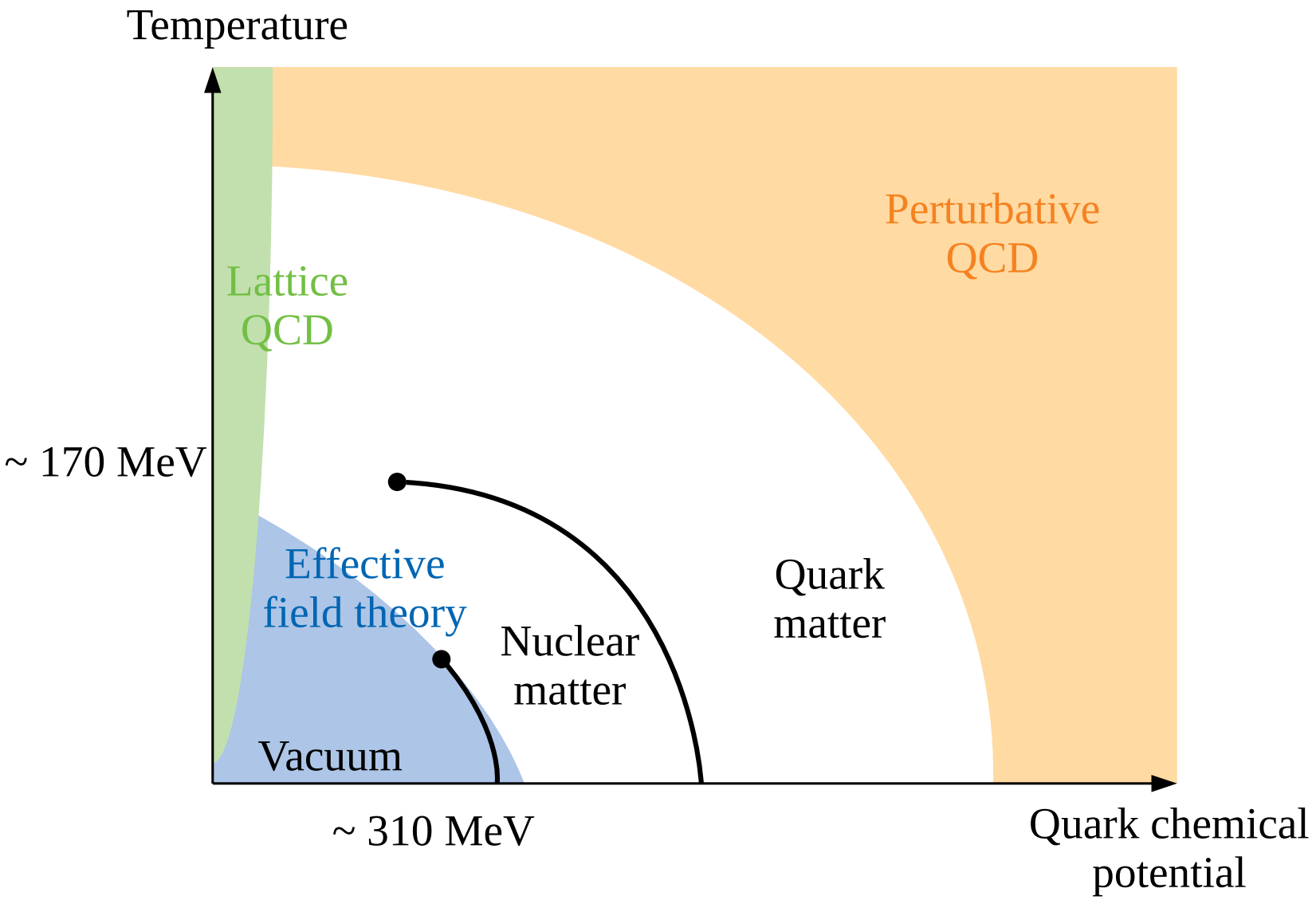

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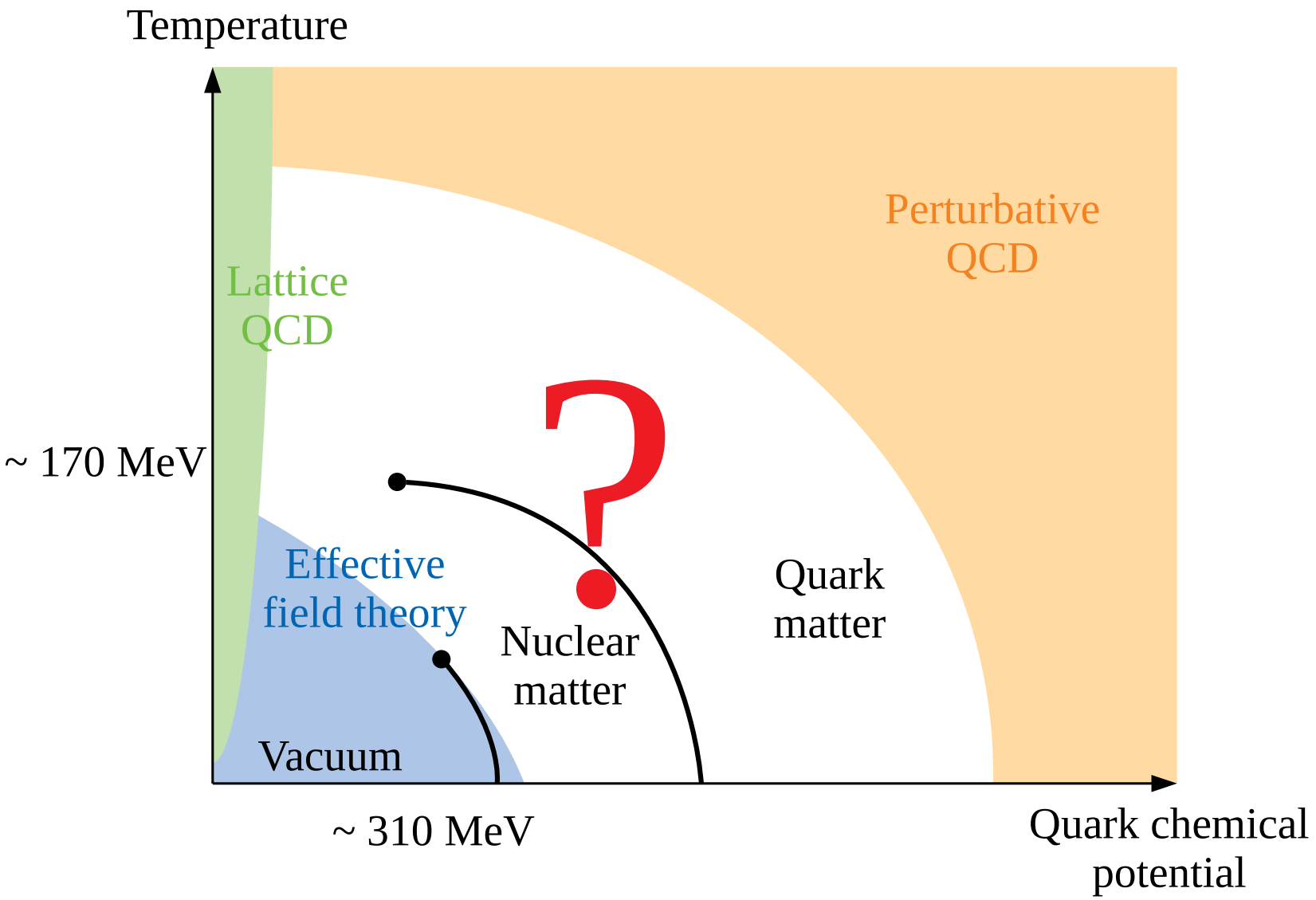

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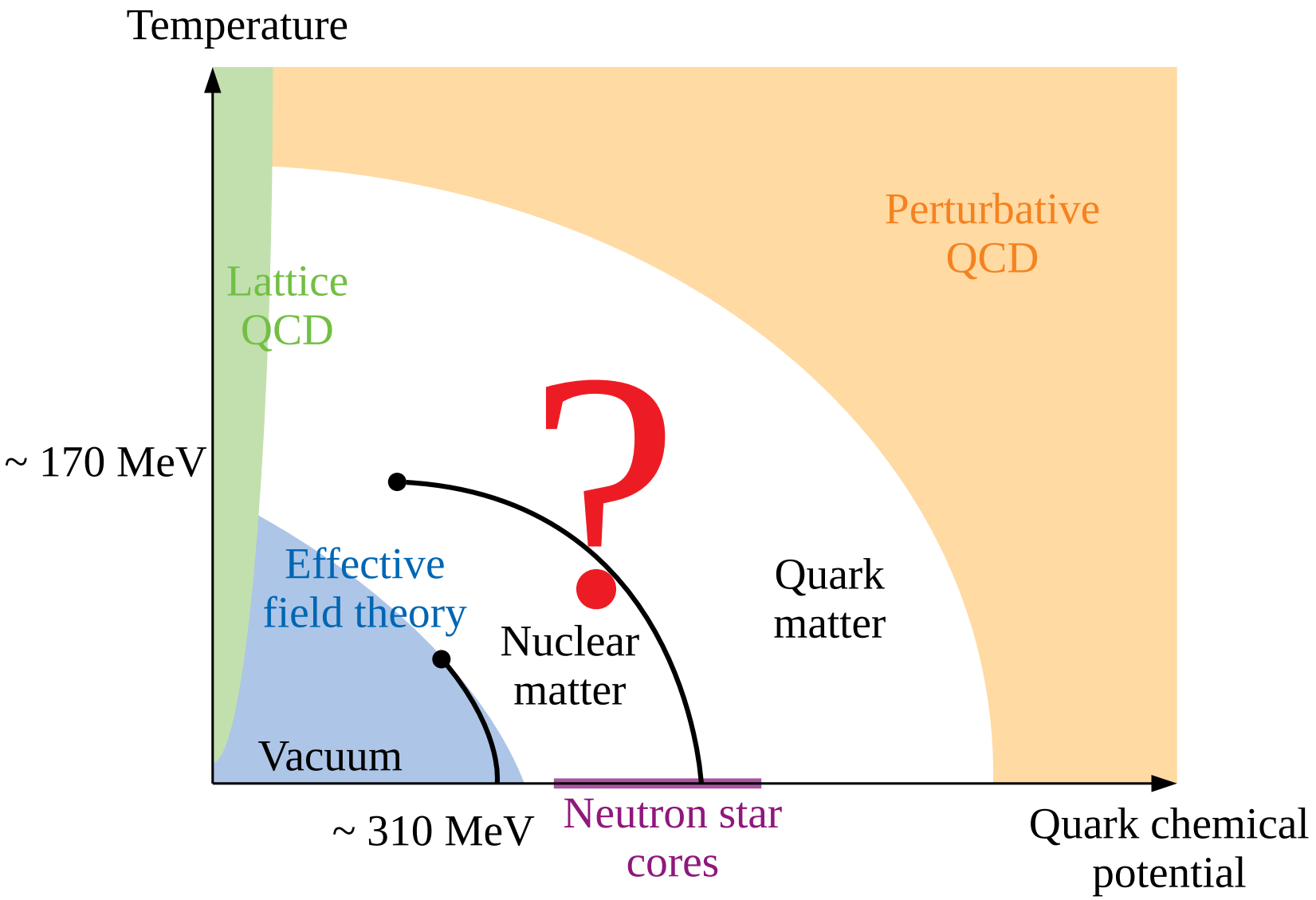

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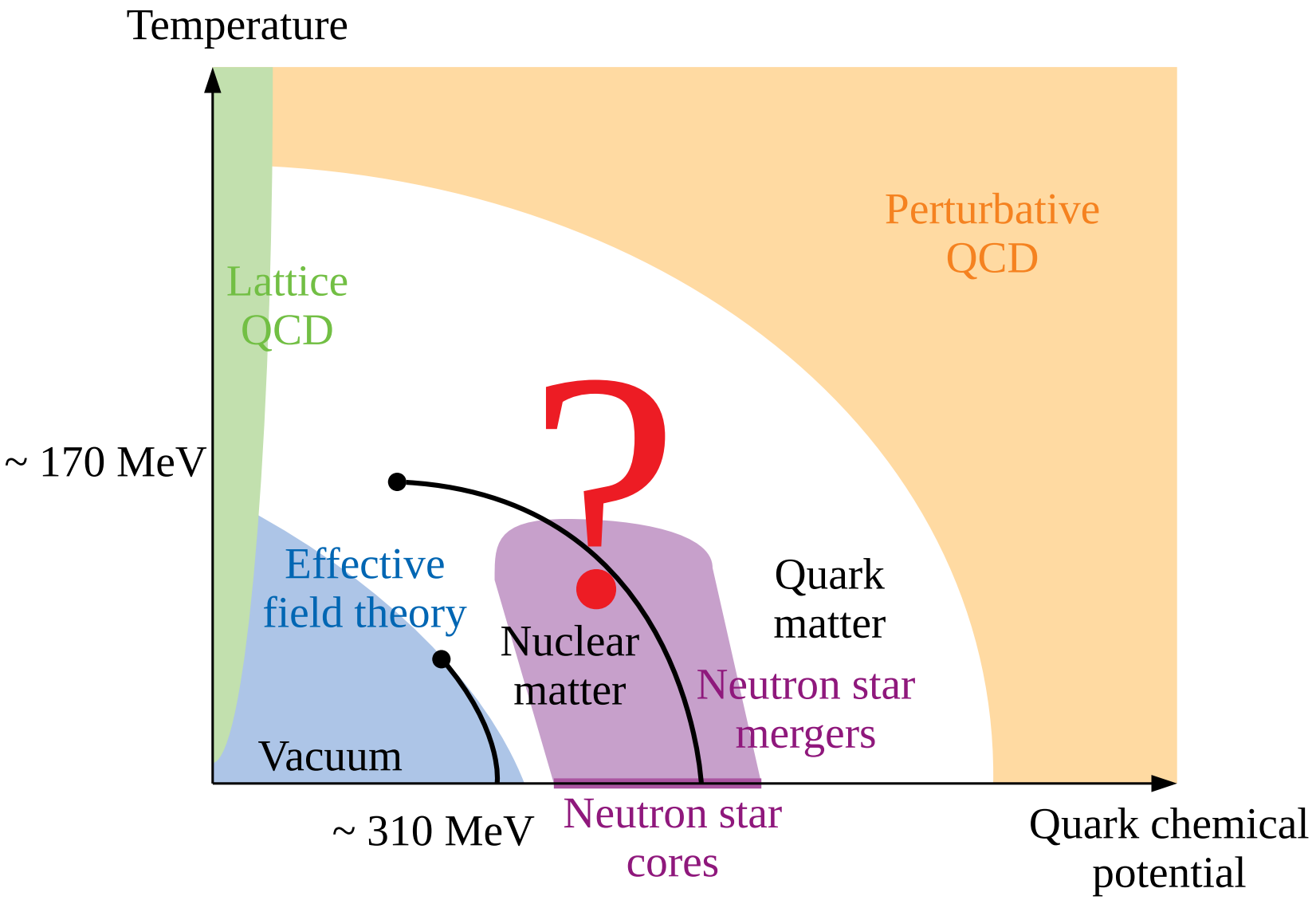


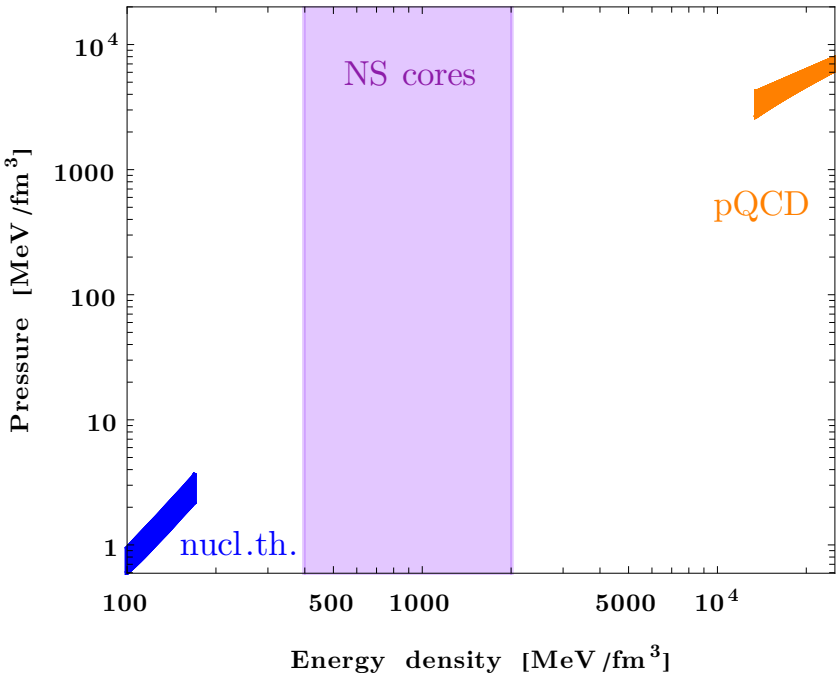
Equation of State

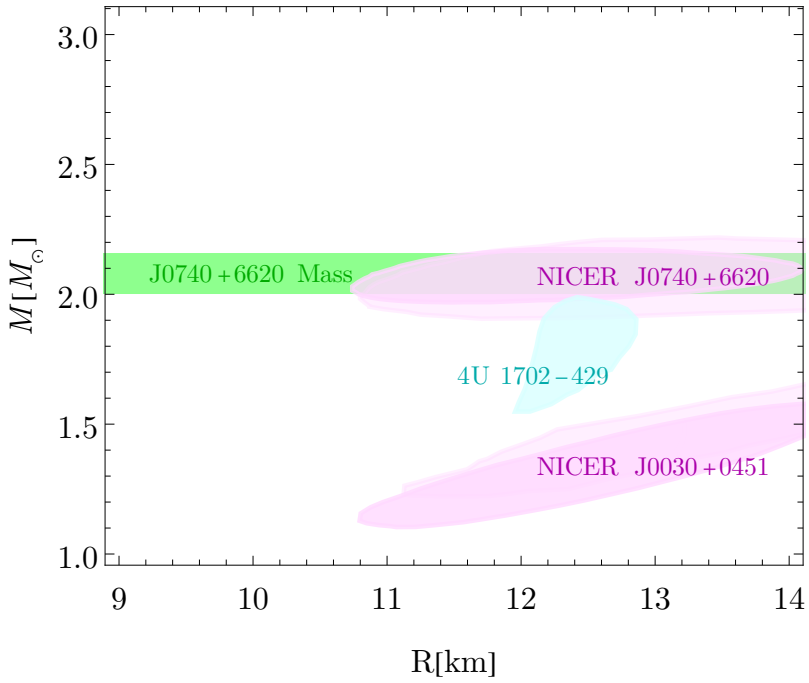
Relates properties of dense matter to properties of neutron stars.

$$p = p(e, \ldots) \xrightarrow{\text{Einstein Eq.}} M(R)$$

Constrained by theory and neutron star observations:

- Causality & themodynamic stability: $0 < c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}e} < 1$
- Direct mass and radius measurements: $M_{\mathrm{TOV}} \gtrsim 2\,M_\odot$, $11\,\mathrm{km} \lesssim R \lesssim 14\,\mathrm{km}$
- First GW detection from a BNS merger GW170817: $\tilde{\Lambda} \lesssim 720$





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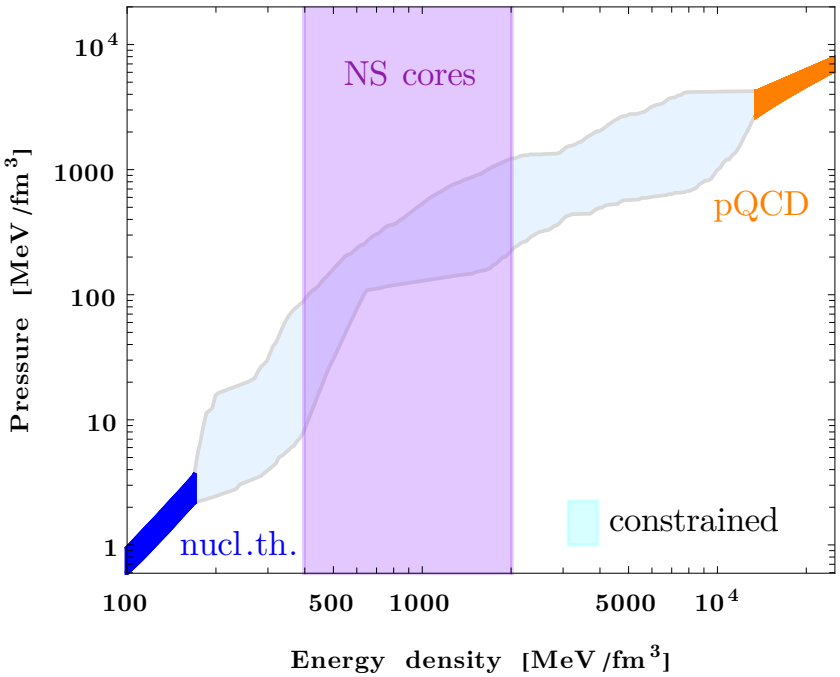
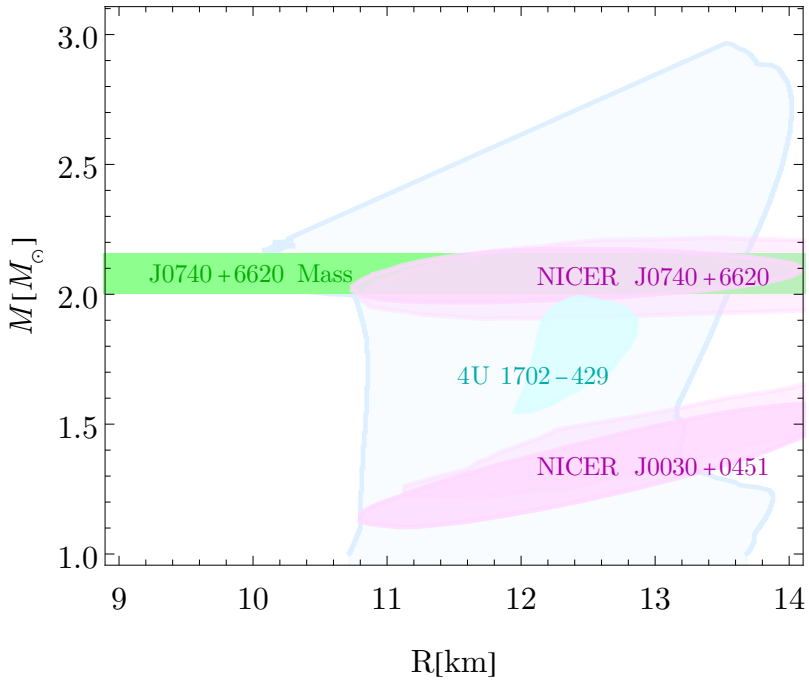
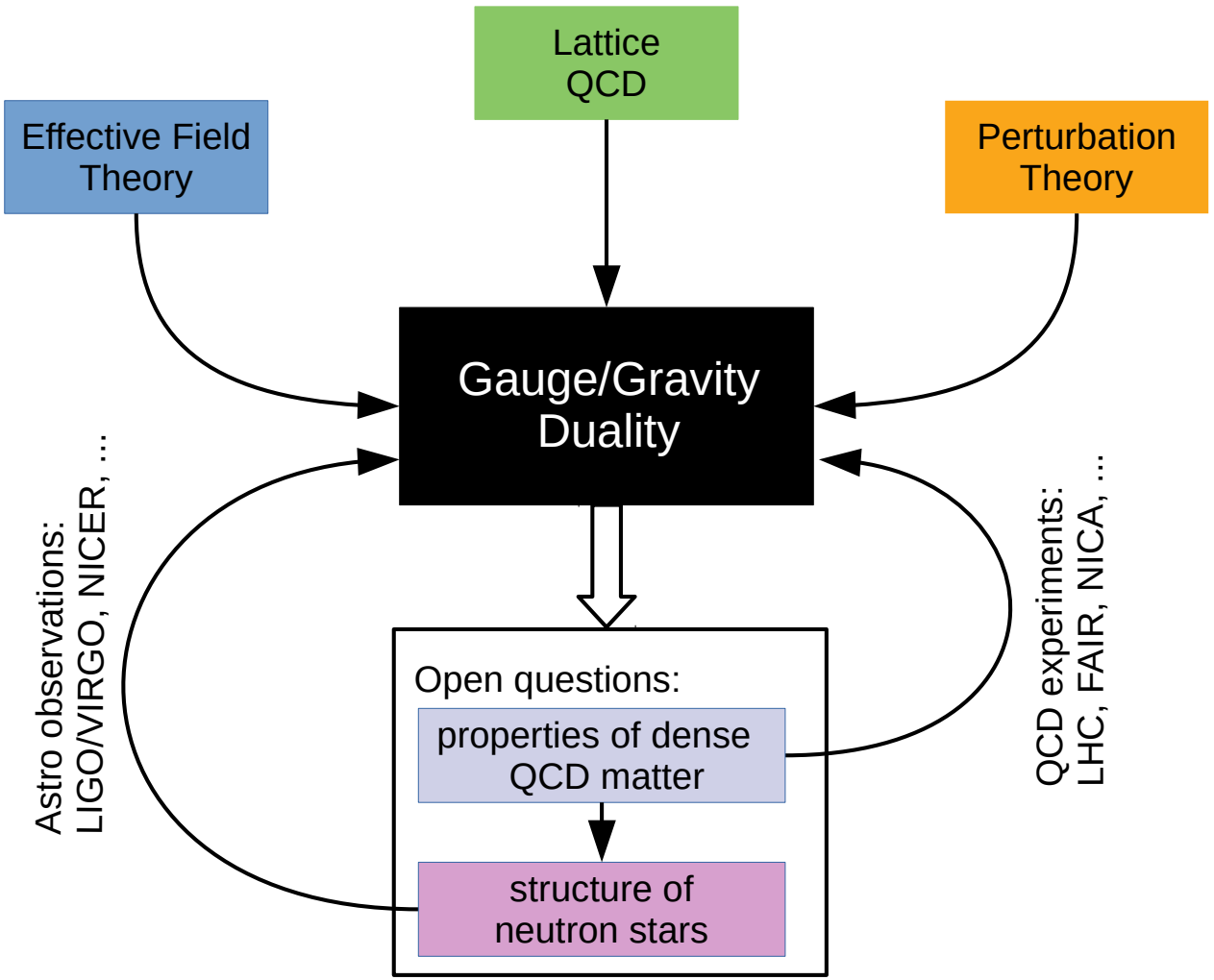




A Holographic Approach to Dense QCD Matter



Holographic Veneziano QCD

Holographic bottom-up model for QCD with many parameters that are tuned to mimic QCD and that are constrained by lattice QCD data.

[Järvinen, Kiritsis arXiv:1112.1261]

Gluons: Improved holographic QCD (Einstein-dilaton gravity)

$$S_{\mathrm{g}} = N_c^2 M^3 \int d^5x \sqrt{-g} \left[R - \tfrac{4}{3} \tfrac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

$\lambda \equiv e^{\phi} \leftrightarrow \mathrm{Tr} F^2$ sources the 't Hooft coupling in YM theory

[Gürsoy, Kiritsis arXiv:0707.1324; Gürsoy, Kiritsis, Nitti arXiv:0707.1349]

Quarks: Tachyonic Dirac-Born-Infeld (DBI) action

$$S_{\mathrm{f}} = -N_f N_c M^3 \int d^5x V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det\left[g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab}\right]}$$

$$F_{rt} = \Phi'(r)\,, \quad \Phi(0) = \mu\,,$$

tachyon $\tau \leftrightarrow \bar{q}q$ controls chiral symmetry breaking.

[Bigazzi et al. arXiv:0505140; Casero et al. arXiv:0702155]

Baryons: homogeneous solution of non-Abelian DBI + Cern–Simons action.

[Ishii, Järvinen, Nijs arXiv:1903.06169]

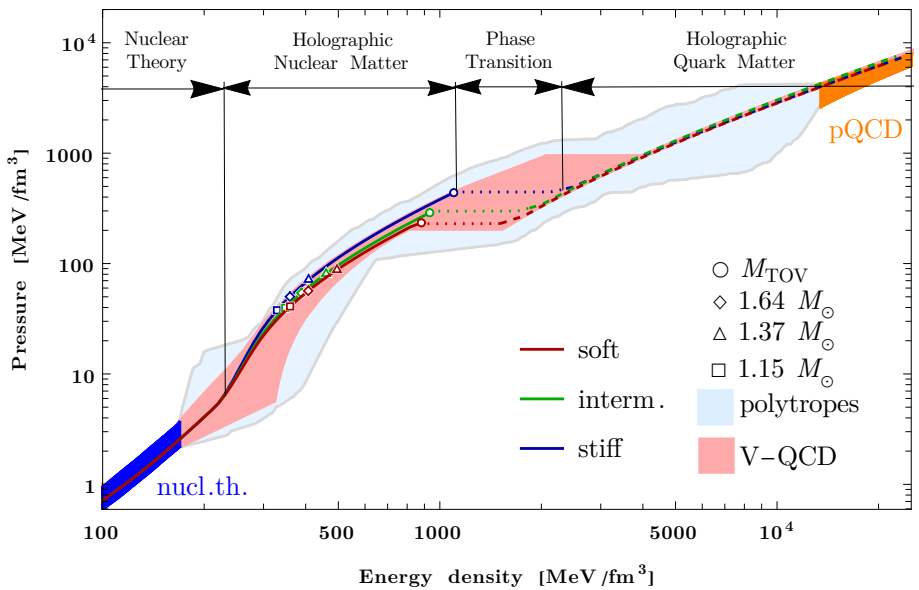
Veneziano limit: $N_c \to \infty$ and $N_f \to \infty$ with $N_f/N_c = \mathcal{O}(1)$ fixed

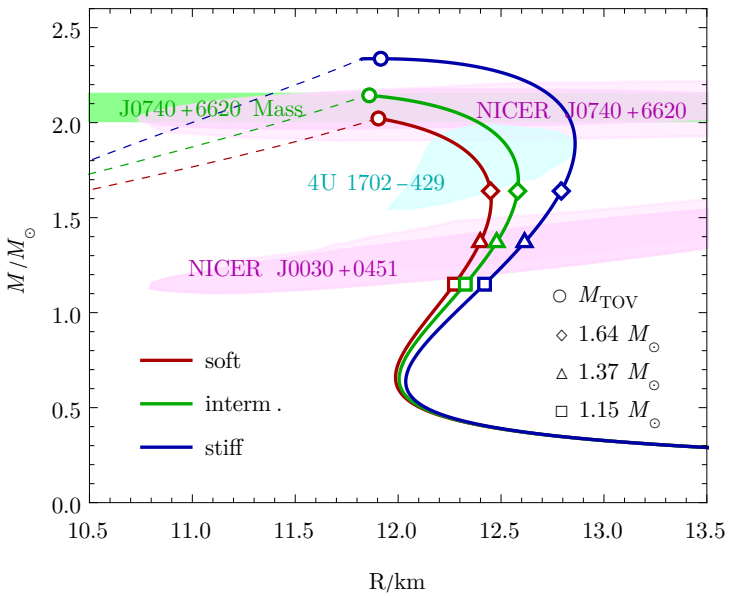
[Järvinen, Kiritsis arXiv:1112.1261]

More details in talk by Matti Järvinen on August 5 2:25 PM (Track G)!

Cold V-QCD Hybrid Equation of State

- To compute neutron star properties we need EoS at low and high densities.
- Homogeneous approximation for V-QCD baryons not reliable at low densities: use results from nuclear theory to model the low density part.
- Hybrid EoSs: nuclear theory model at low densities + V-QCD model for dense baryonic and quark matter at large densities.





[CE, Järvinen, Nijs, van der Schee arXiv:1908.03213; Demircik, CE, Järvinen arXiv:2112.12157]

Finite Temperature Extension

- Temperature dependence is essential when studying BNS-mergers.
- T-dependence of V-QCD baryons is trivial: confining background has no horizon, artefact of large N_c limit.
- Van der Waals construction to extend cold V-QCD baryons to finite-T.
- Ideal gas of protons, neutrons and electrons with excluded volume correction for nucleons.

  $p_{\rm ex}(T, \{\mu_i\}) = p_{\rm id}(T, \{\tilde{\mu}_i\}), \quad \tilde{\mu}_i = \mu_i - v_0 p_{\rm ex}(T, \{\mu_i\}) \quad (i = p,\ n)$
- Add potential term to match with V-QCD at $T = 0$

$$p_{\rm vdW}(T, \{\mu_i\}) = p_{\rm ex}(T, \{\mu_i\}) + \Delta p(\{\mu_i\})$$

$$\Delta p(\{\mu_i\}) = p_{\rm V-QCD}(T = 0, \{\mu_i\}) - p_{\rm ex}(T = 0, \{\mu_i\})$$

[Rischke, Gorenstein, Stoecker, Greiner, Z Phys. C 51, 485 (1991)]

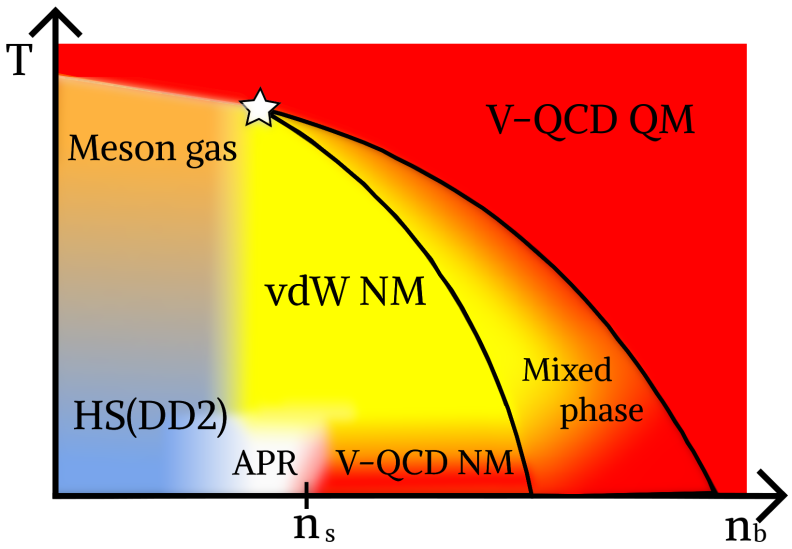
[Vovchenko, Gorenstein, Stoecker, arXiv:1609.03975]

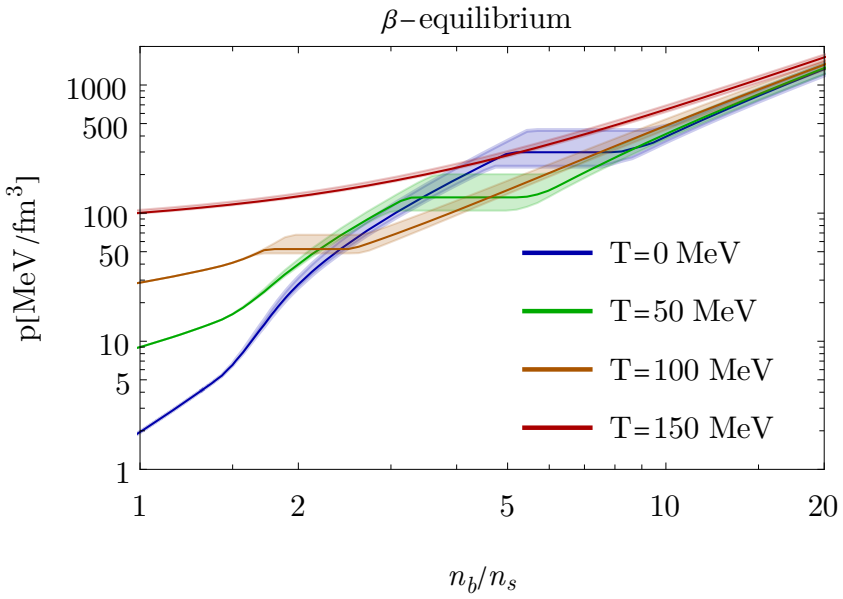
[Vovchenko, Motornenko, Alba, Gorenstein, Satarov, Stoecker, arXiv:1707.09215]

Hot V-QCD Hybrid Equation of State

- Low density part: Hemple-Schaffner-Bielich model (HS)DD2 + Particle Data Group (PDG) mesons.
  [Hempel, Schaffner-Bielich arXiv:0911.4073 ; Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020)]
- (HS)DD2 too stiff around n_s, replace with APR model.
  [Akmal, Pandharipande, Ravenhall arXiv:nucl-th/9804027]
- Mixed baryon and quark matter phase from Gibbs construction

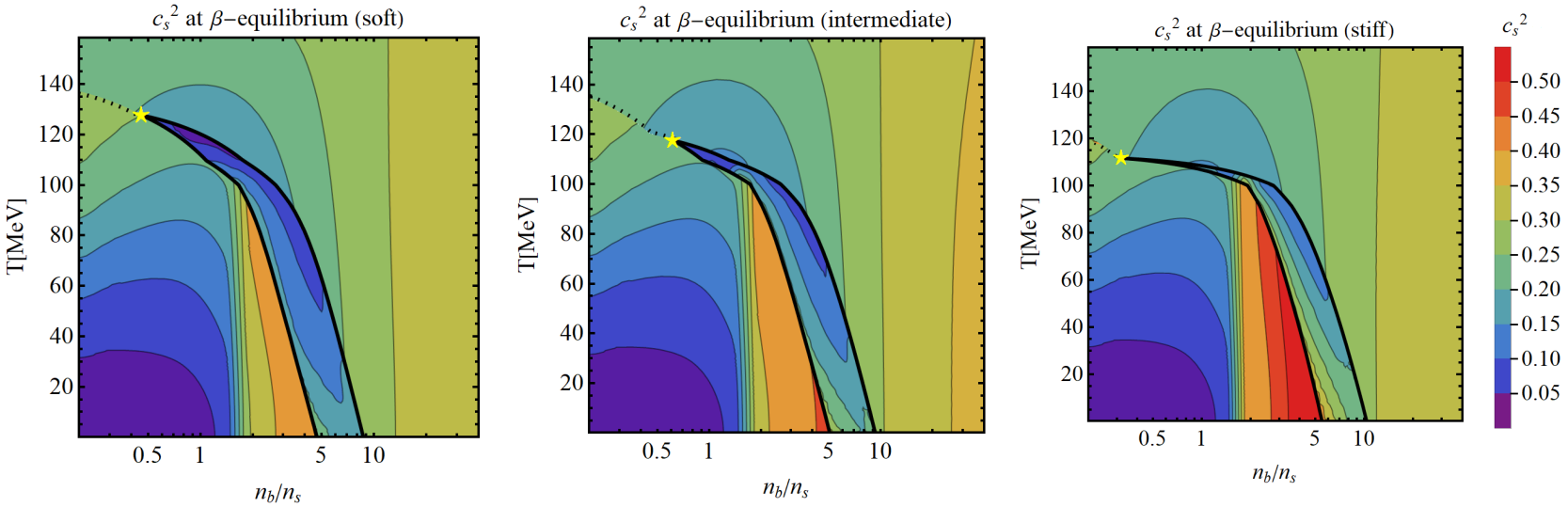
$$X_{NM}(T, n_b^{(1)}, Y_q^{(1)}) = X_{QM}(T, n_b^{(2)}, Y_q^{(2)}), \quad X = \{p, \mu_b, \mu_{le}\}.$$





[Demircik, CE, Järvinen arXiv:2112.12157]

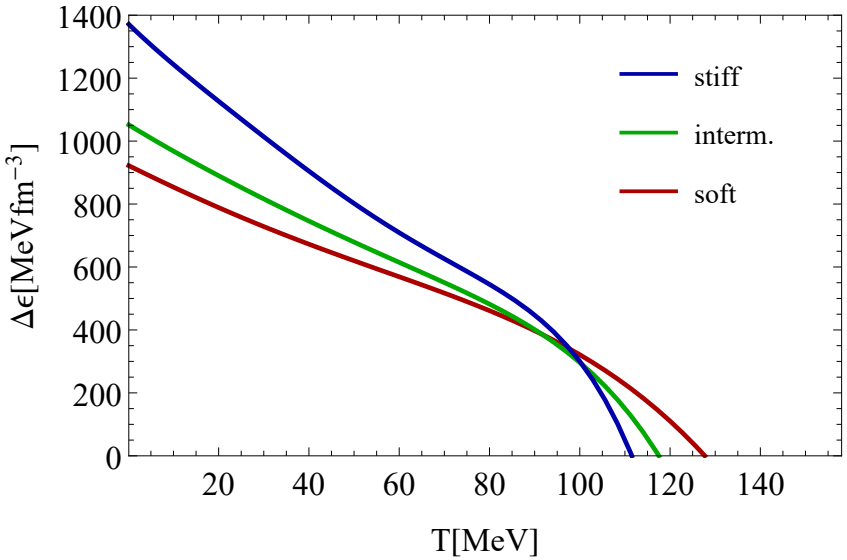
V-QCD Critical Point



Estimates for the critical point:

$$110 \text{ MeV} \lesssim T_c \lesssim 130 \text{ MeV}$$
$$0.3\ n_s \lesssim n_c \lesssim 0.6\ n_s$$

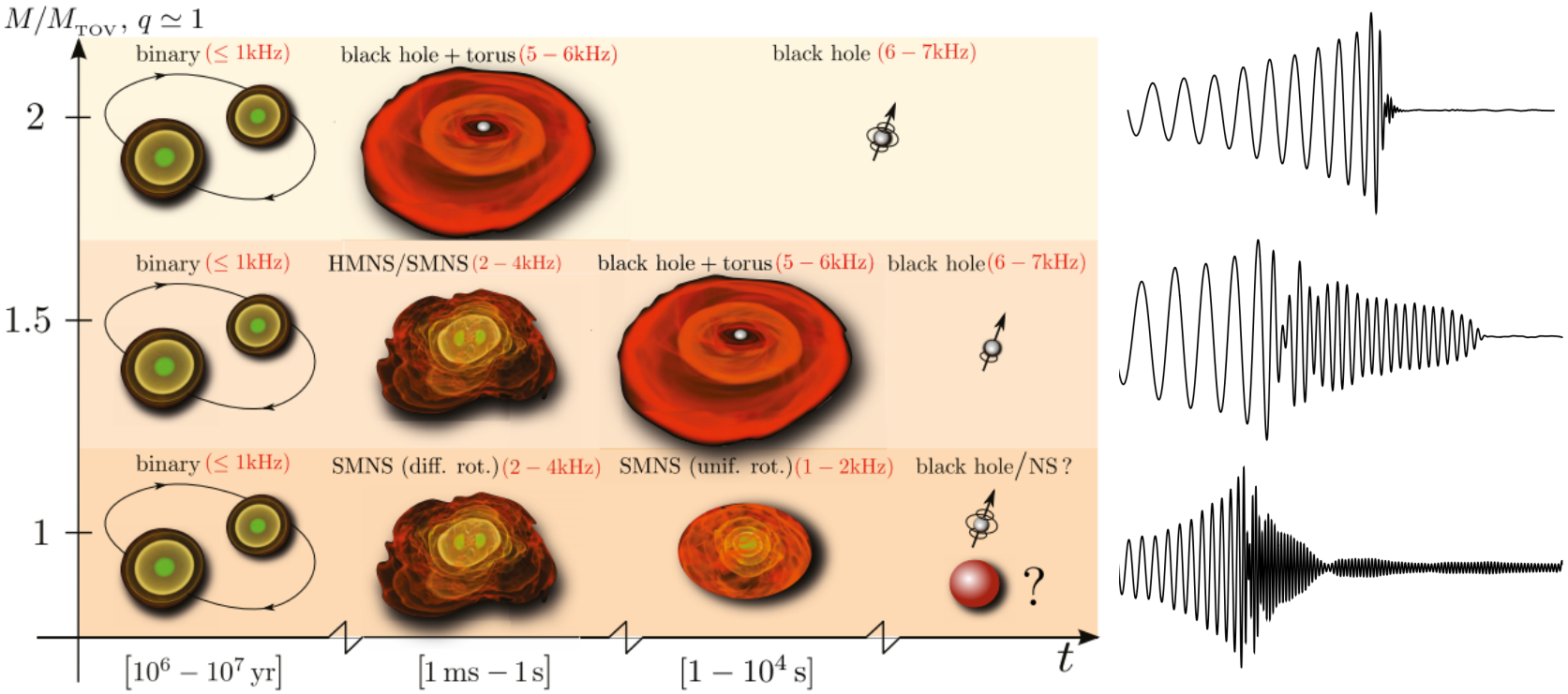
model	$\frac{n_{bc}}{n_s}$	$\frac{\mu_{bc}}{\text{MeV}}$	$\frac{T_c}{\text{MeV}}$
soft	0.46	485	128
interm.	0.62	575	118
stiff	0.32	565	112



[Demircik, CE, Järvinen arXiv:2112.12157]

Binary neutron star mergers

- Significant sources of gravitational radiation.
- Microscopic properties of dense matter encoded in GW and EM signal.
- Likely the origin of short gamma-ray bursts (SGRBs)
- and of heavy elements like gold, platinum, uranium, ...



[picture: Baiotti, Rezzola arXiv:1607.03540]

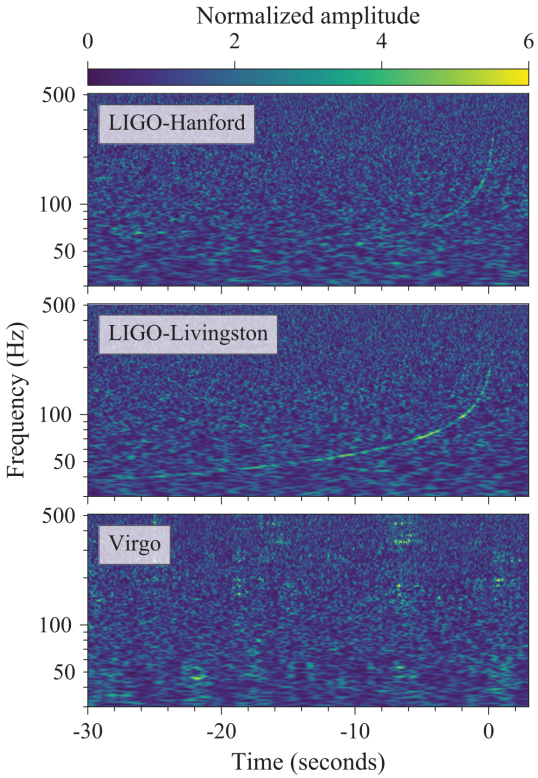
GW170817

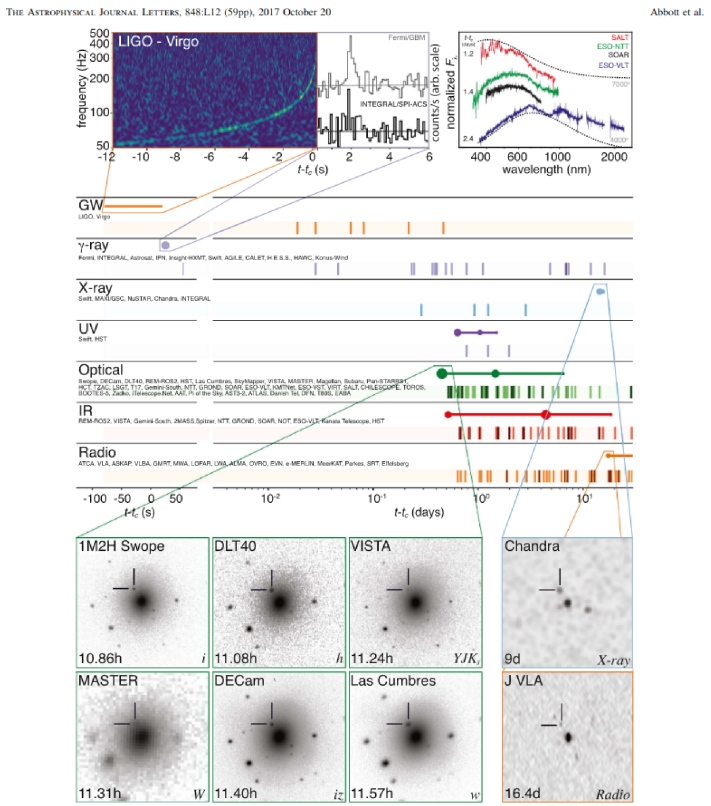
- Best event seen so far: GW, optical, X-ray, γ, UV, IR

  $d \approx 40\,\mathrm{Mpc}\,, \quad \mathcal{M}_{\mathrm{chirp}} = \frac{(M_1 M_2)^{3/5}}{(M1+M2)^{1/5}} = 1.188^{+0.004}_{-0.002}\, M_\odot\,, \quad q = M_2/M_1 > 0.73\,.$
- Inspiral (but no post-merger) waveform: $\tilde{\Lambda} \lesssim 720$

  [LIGO/Virgo: arXiv:1805.11579]
- EM-counterpart: estimates for post-merger HMNS lifetime $\approx 1\,\mathrm{ms}$.

  [Gill, Nathanail, Rezzolla arXiv:1901.04138]





[LIGO/Virgo: arXiv:1710.05832, arXiv:1710.05834]

Simulating Binary Neutron Star Mergers

Have to solve the 3+1D General Relativistic hydrodynamic equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \,, \quad \nabla_\mu T^{\mu\nu} = 0 \,, \quad \nabla_\mu J^\mu = 0$$

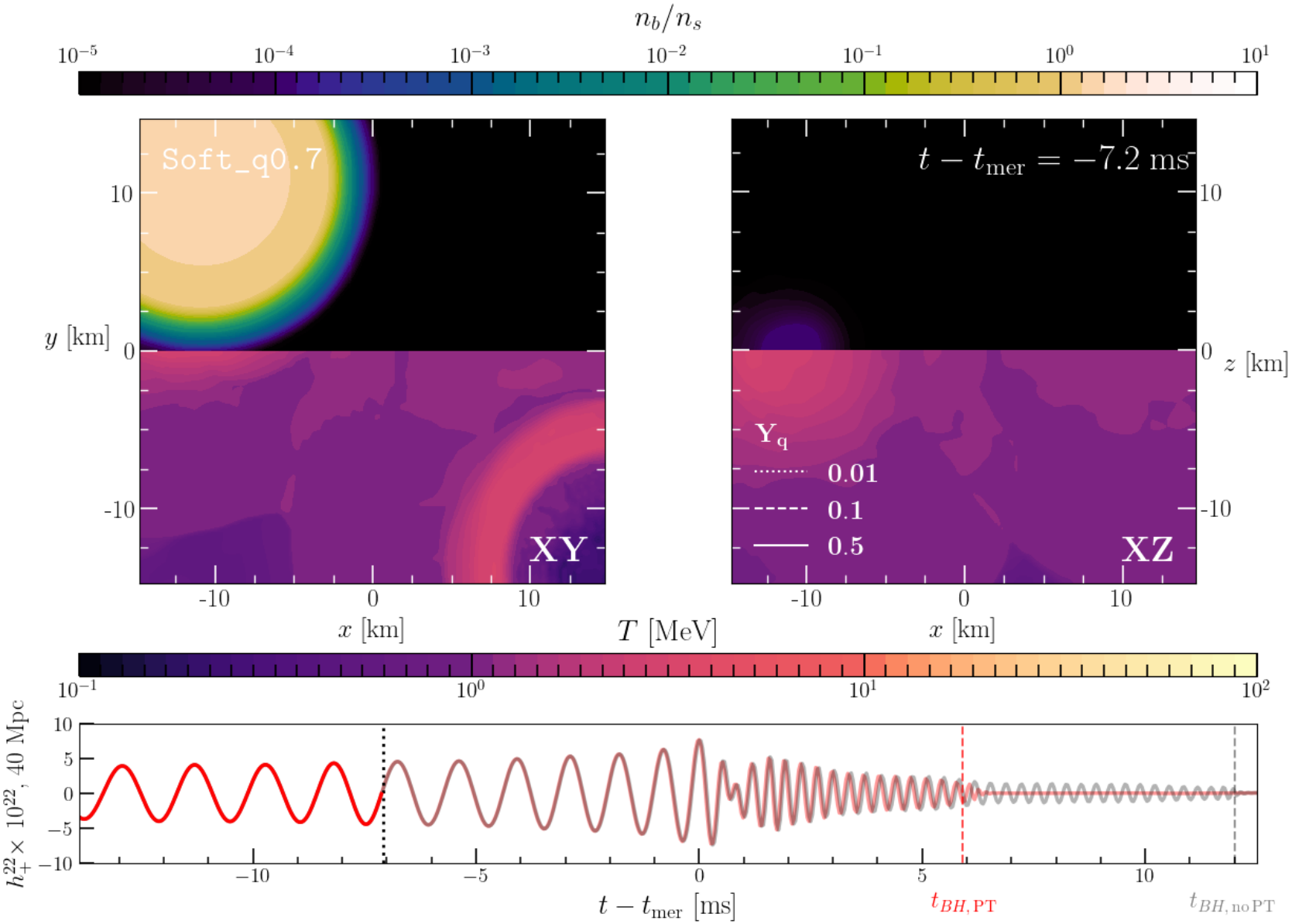
with initial spacetime and fluid distribution modelling a NS binary system.

- Equation of State $p = p(n_b, T, Y_e)$ as input required.
- Spectral code Frankfurt University/Kadath (FUKA) for initial data.
  [Papenfort, Tootle, Grandclement, Most, Rezzolla arXiv:2103.09911]
- Frankfurt/Illinois (FIL) code for binary evolution with tabulated EoS.
  [Most, Papenfort, Rezzolla arXiv:1907.10328]
- Implemented in the Einstein Toolkit.
  [http://einsteintoolkit.org]

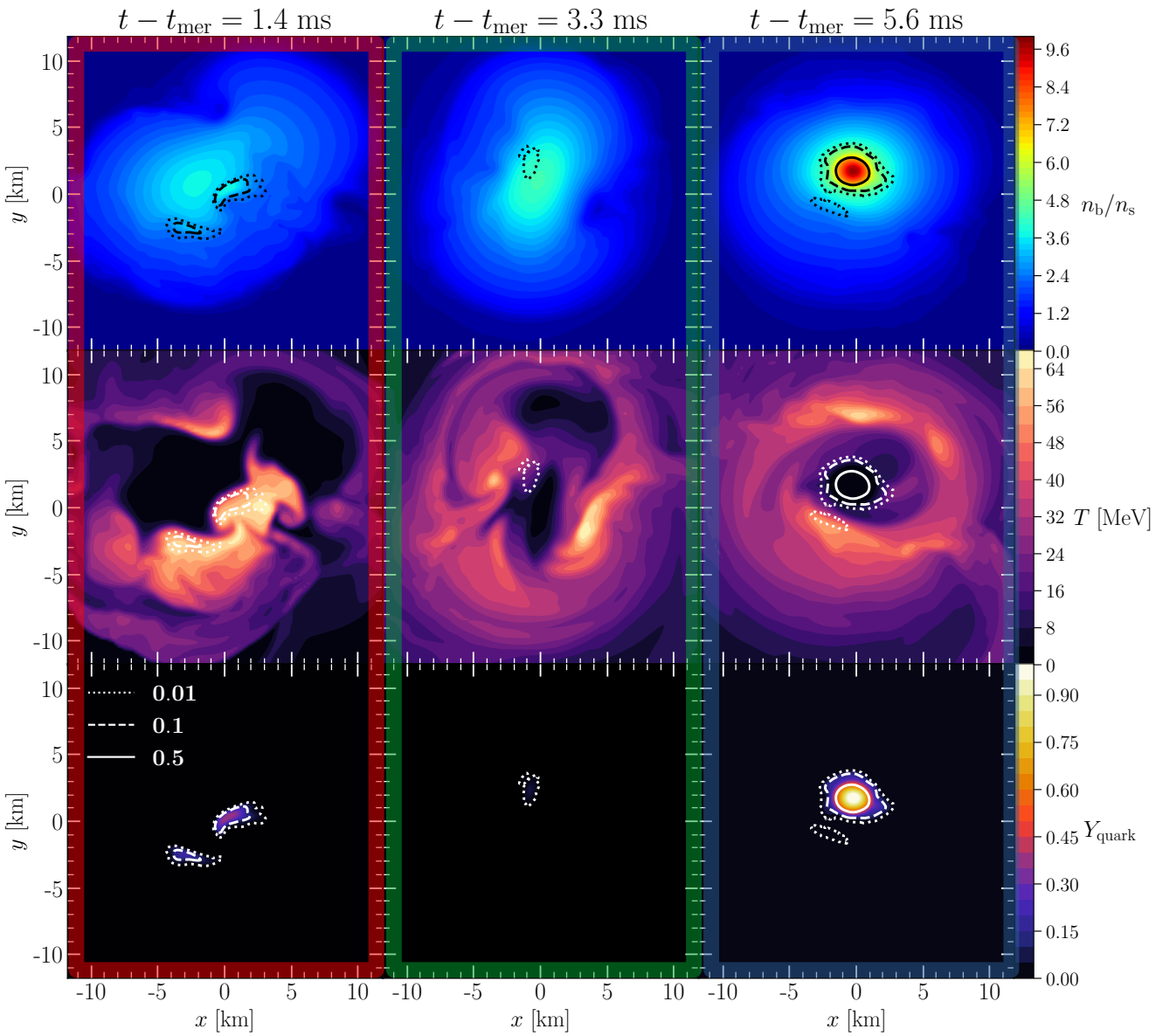
High-Performance Computing Center Stuttgart

- Project BNSMIC with 100+ million core-hours on HAWK.

[Picture Copyright: Ben Derzian for HLRS]

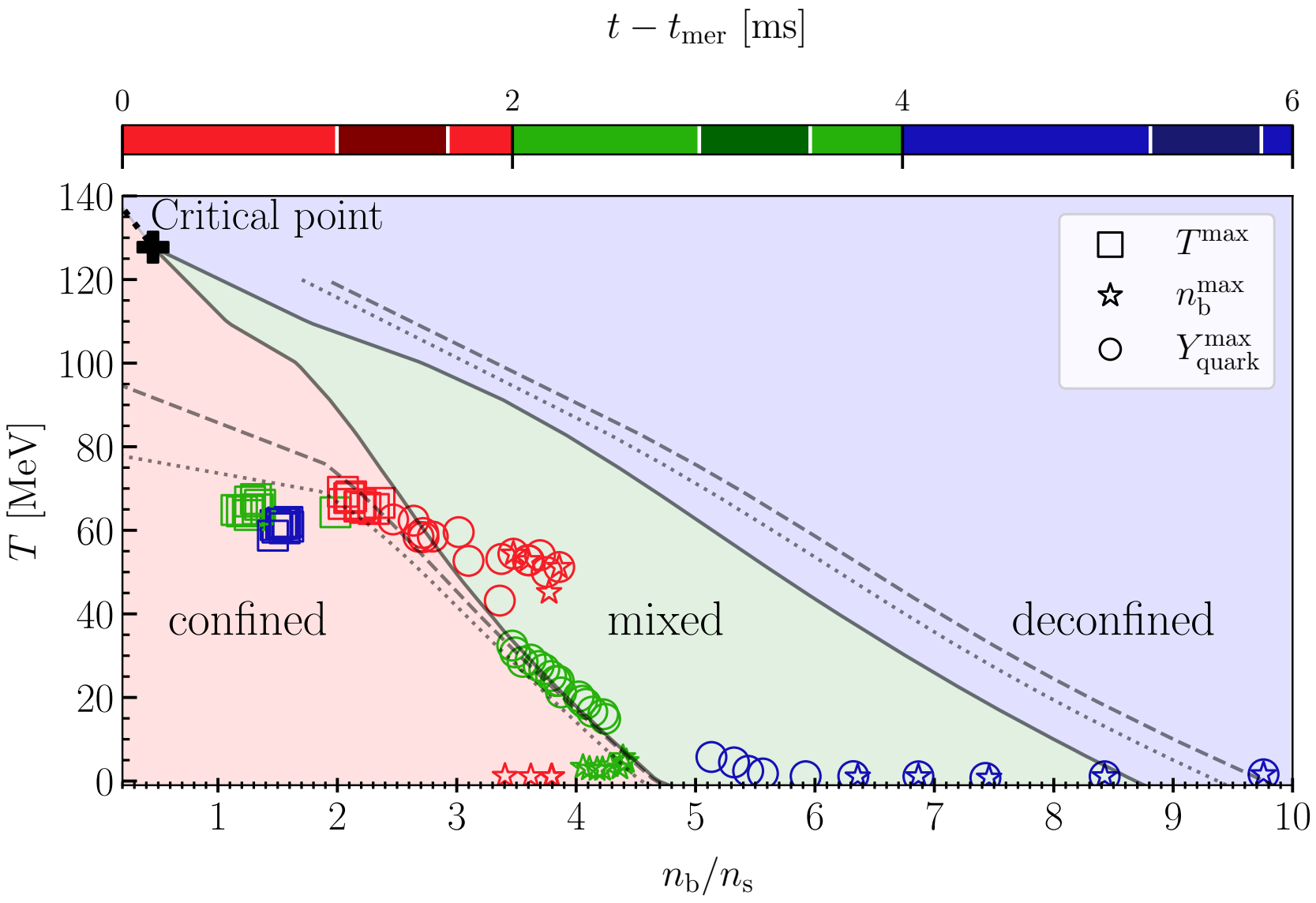

n_b/n_s
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
Soft_q0.7
$t - t_{\rm mer} = -7.2$ ms
y [km]
z [km]
x [km]
XY
XZ
Y_q
0.01
0.1
0.5
T [MeV]
10^{-1} 10^{0} 10^{1} 10^{2}
$h_+^{22} \times 10^{22}$, 40 Mpc
$t - t_{\rm mer}$ [ms]
$t_{BH,\rm PT}$
$t_{BH,\rm no\,PT}$

Hot, Warm and Cold Quarks



[Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla arXiv:2205.05691]

Phase Diagram



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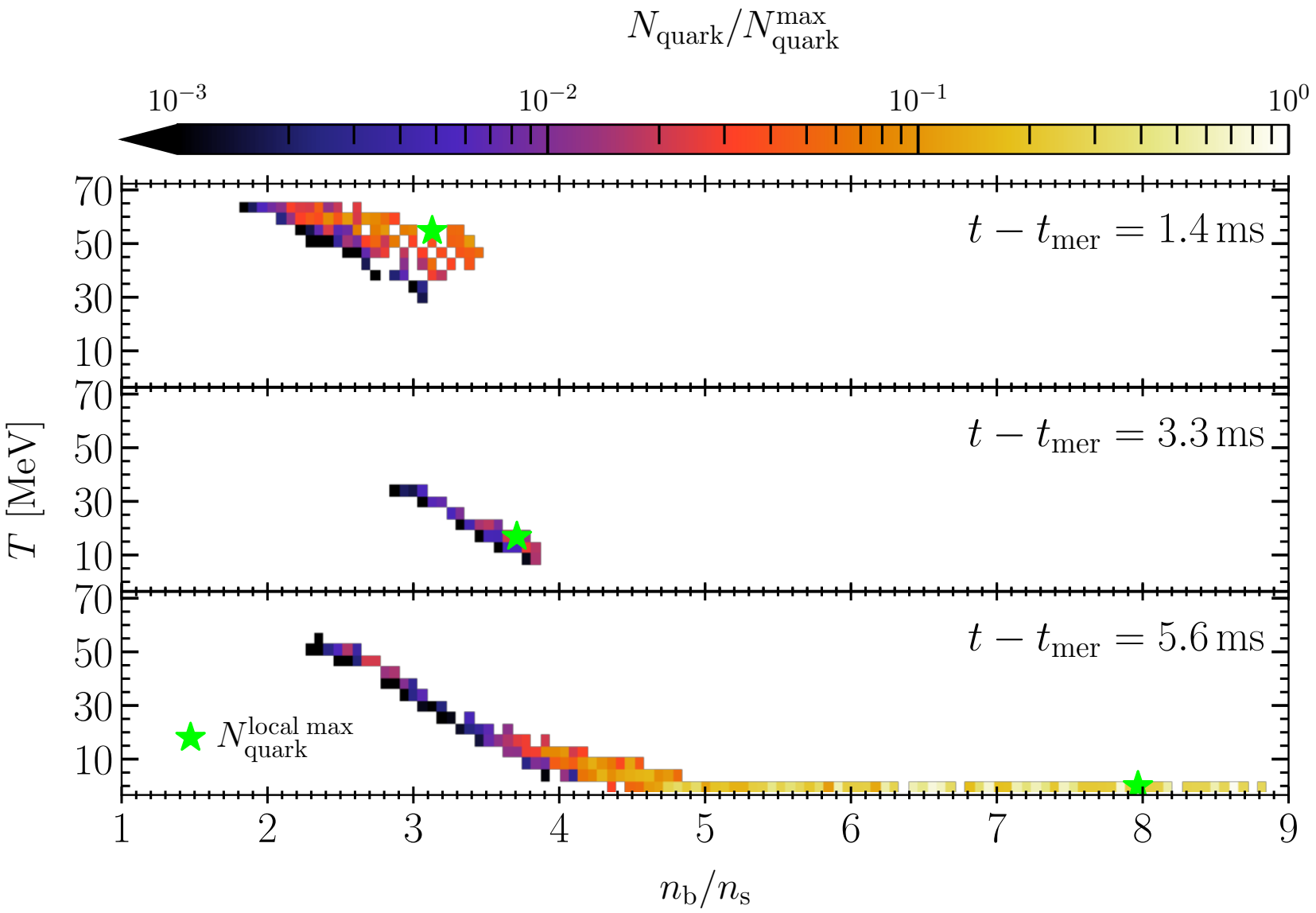
Quark Abundance



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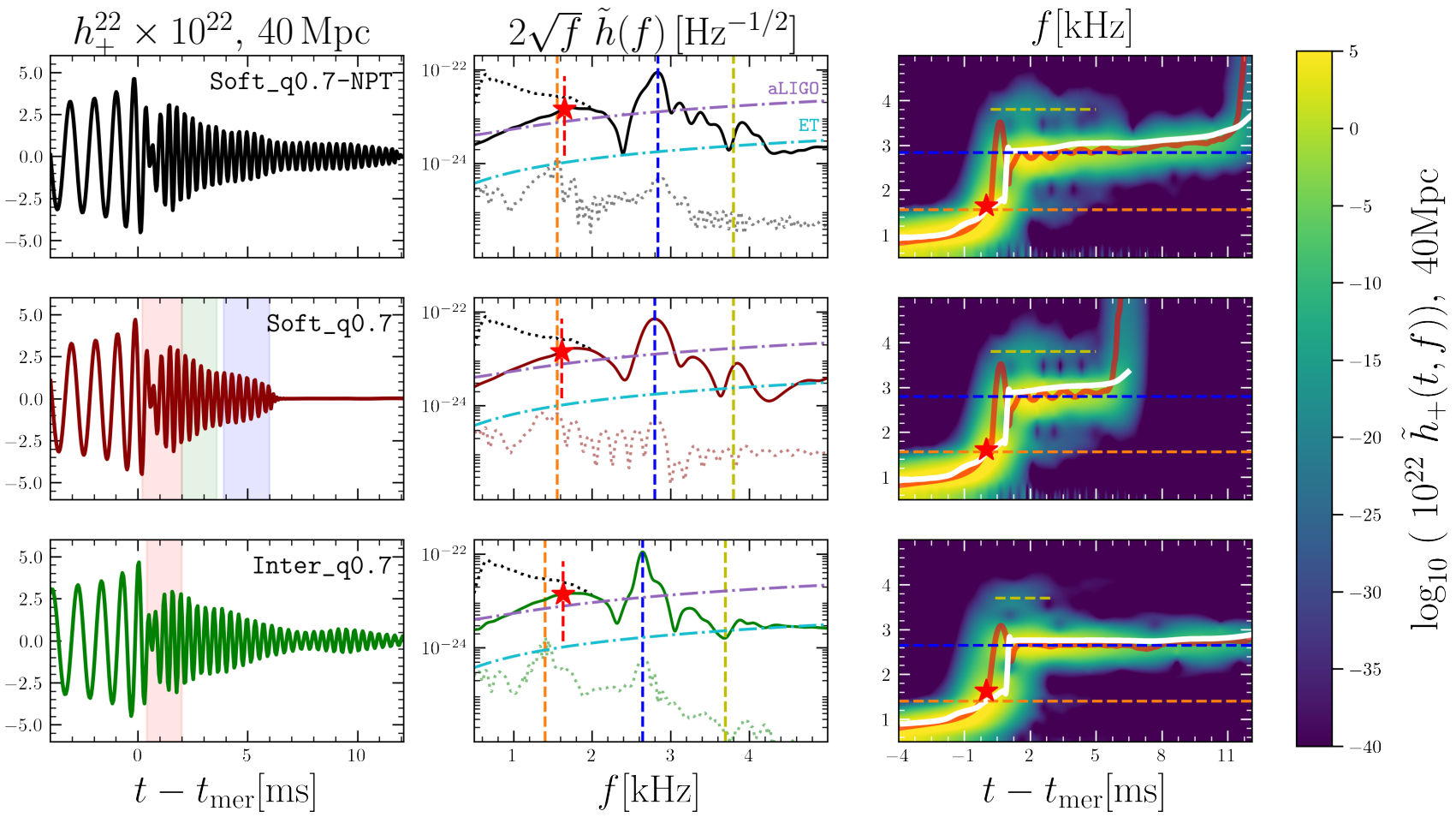
Waveforms and Frequency Spectra

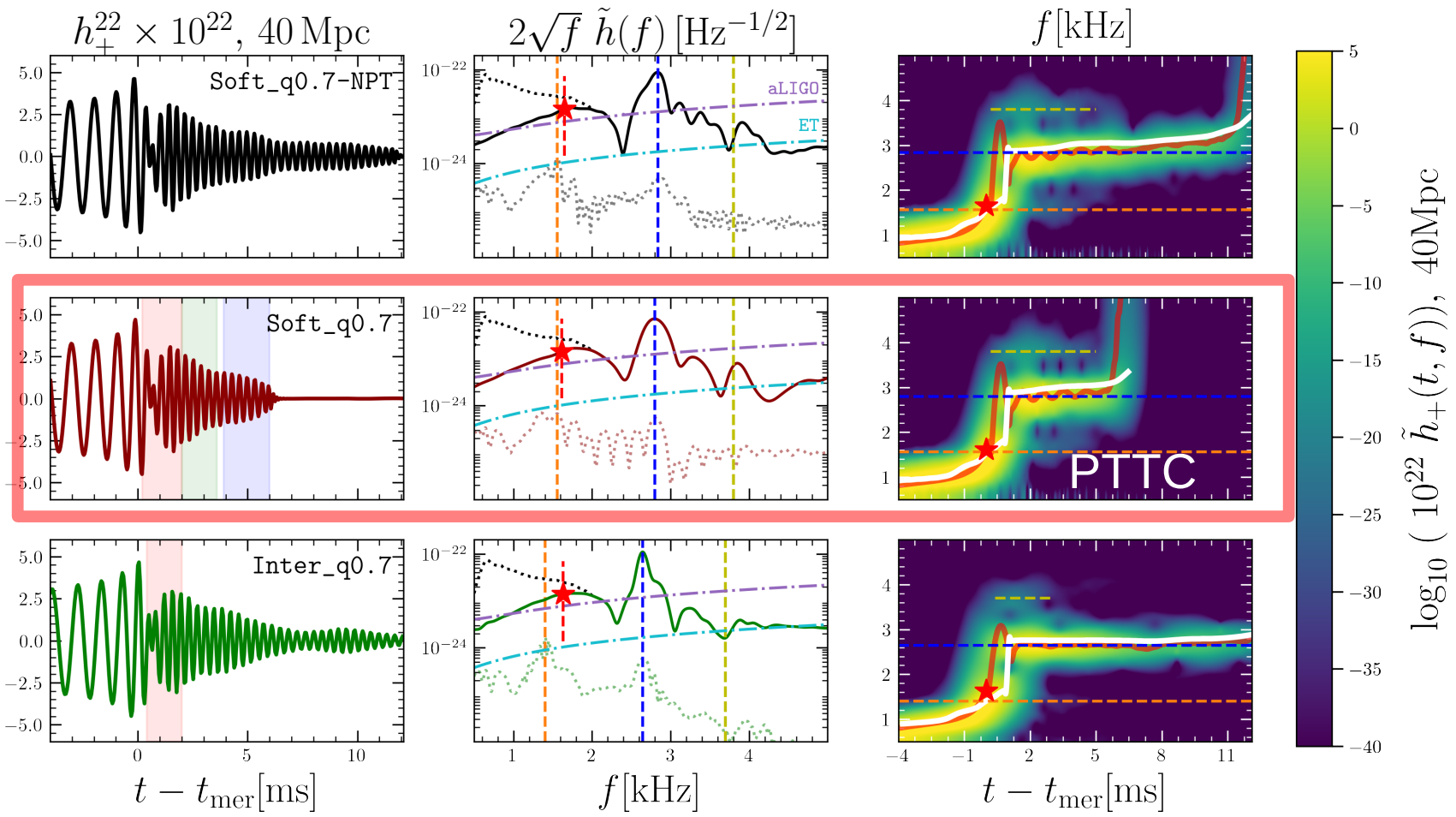
- Small impact of quark matter on post-merger frequencies.



[Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla arXiv:2205.05691]

Waveforms and Frequency Spectra

- Small impact of quark matter on post-merger frequencies.
- PTTC leads to shorter lifetime of the hyper massive neutron star (HMNS).



[Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla arXiv:2205.05691]

Summary

- Hot V-QCD hybrid EoS with deconfinement phase transition, three tabulated versions in CompOSE format (stay tuned).
- Predictions for the QCD critical point:

$$110\ \mathrm{MeV} \lesssim T_c \lesssim 130\ \mathrm{MeV}\,, \quad 0.3\ n_s \lesssim n_c \lesssim 0.6\ n_s\,.$$

[Demircik, CE, Järvinen arXiv:2112.12157]

- Neutron star merger simulations with V-QCD EoS.
- Identified 3 different stages in HMNSs: hot, warm, cold quark matter.
- Small impact on frequencies, but PTTC shortens HMNS lifetime.

[Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla arXiv:2205.05691]

Ongoing work:

- Prompt collapse: Imprint of quark formation on threshold mass?
- Long-time stability (lifetime) of HMNS: Need simulations up to 1-2 sec. (expensive, simplified spacetime evolution, axial-symmetry, ...)
- ...

V-QCD without baryons (I)

Consider first the non-baryonic V-QCD action, whose solutions will serve as background for the probe baryons

$$S^{(0)}_{\mathrm{V-QCD}} = S_{\mathrm{glue}} + S^{(0)}_{\mathrm{DBI}}\,.$$

The gluon part is given by the IHQCD (dilaton gravity) action

$$S_{\mathrm{glue}} = N_c^2 M^3 \int d^5x \sqrt{-g}\left[R - \tfrac{4}{3}\tfrac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right],$$

where $\lambda \equiv e^{\phi} \leftrightarrow \mathrm{Tr}F^2$ ($\approx g^2 N_c$ near the boundary) sources the 't Hooft coupling in YM theory, the dilaton potential is chosen[1] to mimic QCD

$$V_g(\lambda) = 12\left[1 + V_1\lambda + \tfrac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\mathrm{IR}} e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)}\right].$$

Finite T is implemented by homogeneous+isotropic black brane metric

$$ds^2 = e^{2A(r)}(-f(r)dt^2 + d\vec{x}^2 + f^{-1}(r)dr^2)\,.$$

[1] E.g. V_1 and V_2 are fixed by requiring the UV RG flow of the 't Hooft coupling to be the same as in QCD up to two-loop order.

V-QCD without baryons (II)

The flavor part is modelled by the tachyonic DBI-action[2]

$$S^{(0)}_{\rm DBI} = -N_f N_c M^3 \int d^5x V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det\left[g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab}\right]}\,,$$

$$F_{rt} = \Phi'(r)\,, \quad \Phi(0) = \mu\,,$$

where the tachyon $\tau \leftrightarrow \bar{q}q$ controls chiral symmetry breaking.

Several potentials: $\{V_g(\lambda), V_{f0}(\lambda), w(\lambda), \kappa(\lambda)\}$, chosen to match pQCD in UV ($\lambda \to 0$), qualitative agreement with QCD in IR ($\lambda \to \infty$) and tuned to lattice QCD in the middle ($\lambda \sim \mathcal{O}(1)$).

[For details see Appendix B of Ishii, Järvinen, Nijs arXiv:1903.06169]

Different solutions:

without/with horizon $\leftrightarrow$ confined/deconfined phase
without/with tachyon $\leftrightarrow$ chirally symmetric/chirally broken phase

[2] Without baryons we have a vectorial flavor singlet gauge field $A^{(L/R)} = \mathbb{I}_f \Phi(r) dt$ giving nonzero charge density and chemical potential.

Probe baryons in V-QCD

Each baryon maps to a solitonic "instanton" configuration of non-Abelian gauge fields in the bulk.

[Witten; Gross, Ooguri; ...]

Consider the full non-Abelian brane action $S = S_{\rm DBI} + S_{\rm CS}$ where

[Bigazzi, Casero, Cotrone, Kiritsis, Paredes; Casero, Kiritsis, Paredes]

$$S_{\rm DBI} = -\frac{1}{2} M^3 N_c \, \mathbb{T}r \int d^5x \, V_{f0}(\lambda) e^{-\tau^2} \left(\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}} \right) ,$$

$$\mathbf{A}^{(L/R)}_{MN} = g_{MN} + \delta^r_M \delta^r_N \kappa(\lambda) \tau'(r)^2 + \delta^{rt}_{MN} w(\lambda) \Phi'(r) + w(\lambda) F^{(L/R)}_{MN}$$

gives the dynamics of the solitons.
The Cern-Simons term sources the baryon number for the solitions

$$S_{\rm CS} = \frac{N_c}{8\pi^2} \int \Phi(r) e^{-b\tau^2} dt \wedge \left(F^{(L)} \wedge F^{(L)} - F^{(R)} \wedge F^{(R)} + \cdots \right) .$$

Non-Abelian DBI action only known to first few orders in $F^{(L/R)}$: expand to second order on top of solution $(g_{MN}, \Phi, \lambda, \tau)$ obtained from $S^{(0)}_{V-QCD}$.

Homogeneous Baryon Ansatz

Set $N_f = 2$ and consider the SU(2) Ansatz

[Rozali, Shieh, Van Raamsdonk, Wu]

$$A_L^i = -A_R^i = h(r)\sigma^i$$

Immediate consequence: baryon charge integrates to zero?

$$N_b \propto \int dr \frac{d}{dr}\left[e^{-b\tau^2} h^3 (1 - 2b\tau^2)\right] \stackrel{?}{=} 0$$

However finite baryon number may can be realized by discontinuity of h $\leftrightarrow$ smeared solitons in singular gauge.

[Ishii, Järvinen, Nijs, arXiv:1903.06169]

The free parameter b of the model is used to tune the baryon onset to its physical value in QCD.

Binary tidal deformability and chirp mass

- Chirp mass can be extracted accurately from inspiral GW strain signal

$$\mathcal{M}_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = \left(\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right)^{3/5} . \tag{1}$$

- Binary tidal deformability encodes EOS properties via Λ_i

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12M_2 + M_1)\, M_1^4 \Lambda_1 + (12M_1 + M_2)\, M_2^4 \Lambda_2}{(M_1 + M_2)^5} . \tag{2}$$

- GW170817 only "good" event so far:

$$\mathcal{M}_{chirp} = 1.188^{+0.004}_{-0.002} M_\odot, \quad q = M_2/M_1 > 0.7, \quad \tilde{\Lambda} \lesssim 720 . \tag{3}$$

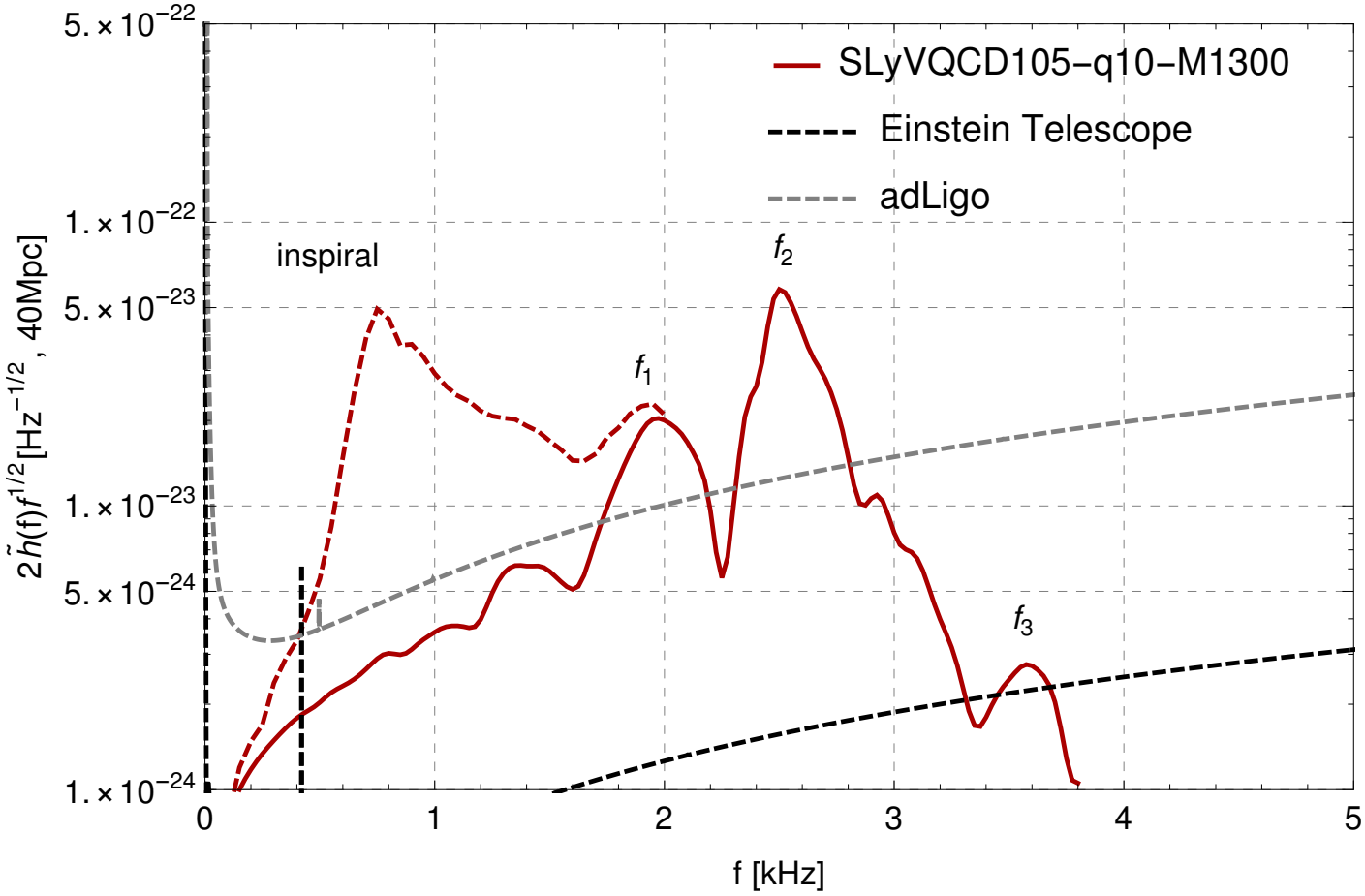
[LIGO/Virgo: arXiv:1805.11579]

Power Spectral Density

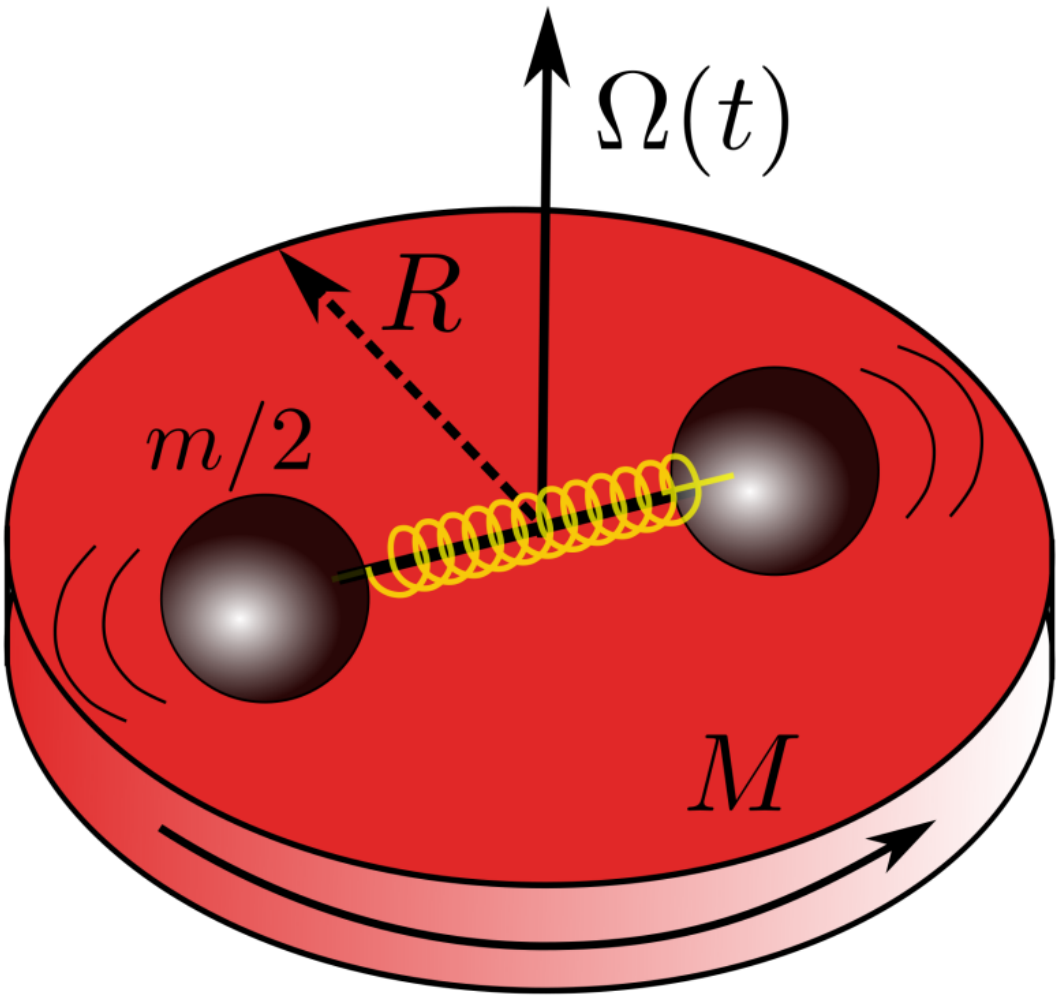
Post-merger power spectral density (PSD) has typical three peak structure

$$\tilde{h}(f) \equiv \sqrt{\frac{|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2}{2}}\,, \quad \tilde{h}_{+,\times}(f) \equiv \int h_{+,\times}(t) e^{-i2\pi f t} dt\,.$$

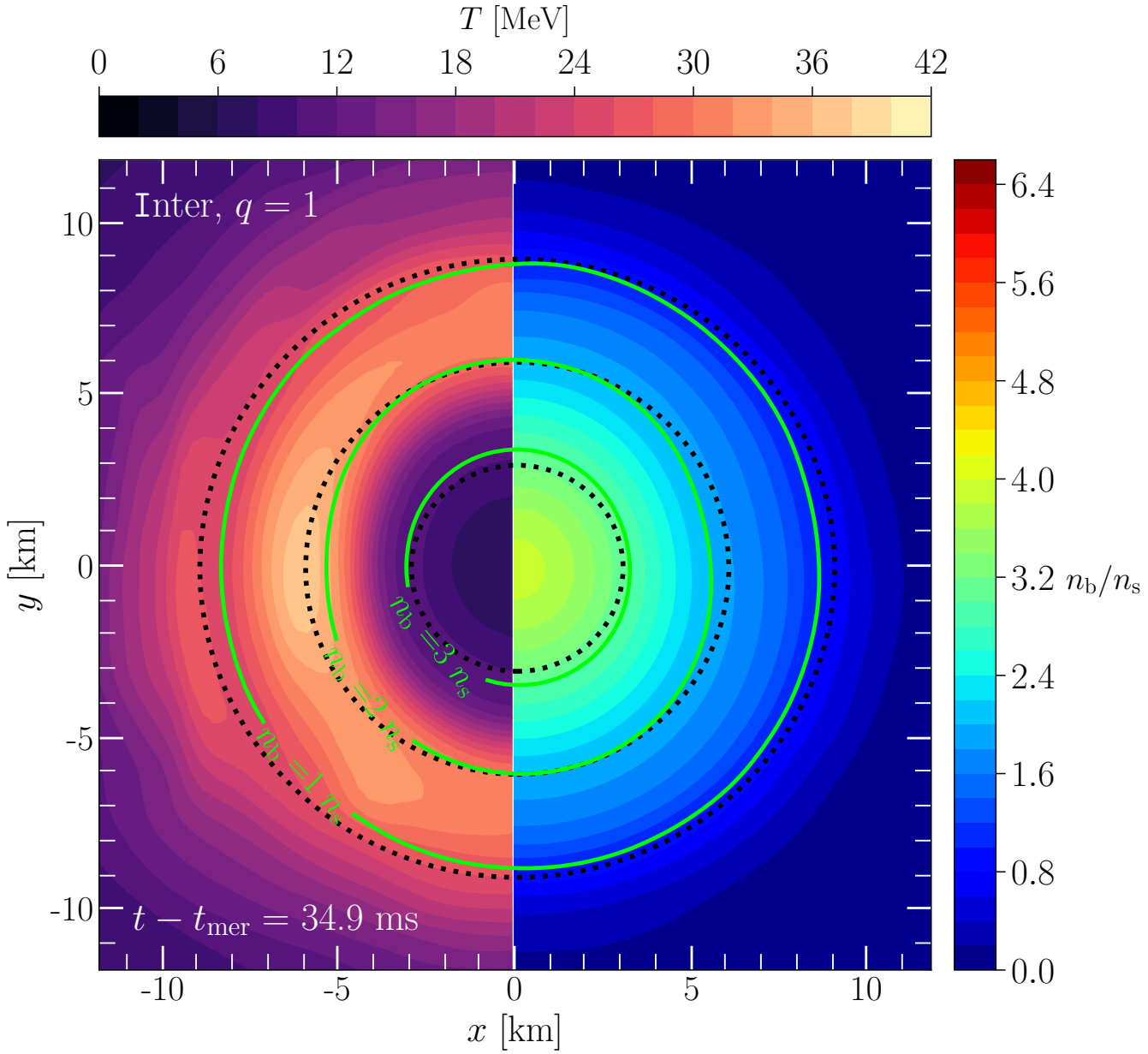
Characteristic frequencies f_1, f_2, f_3 encode information about EoS.

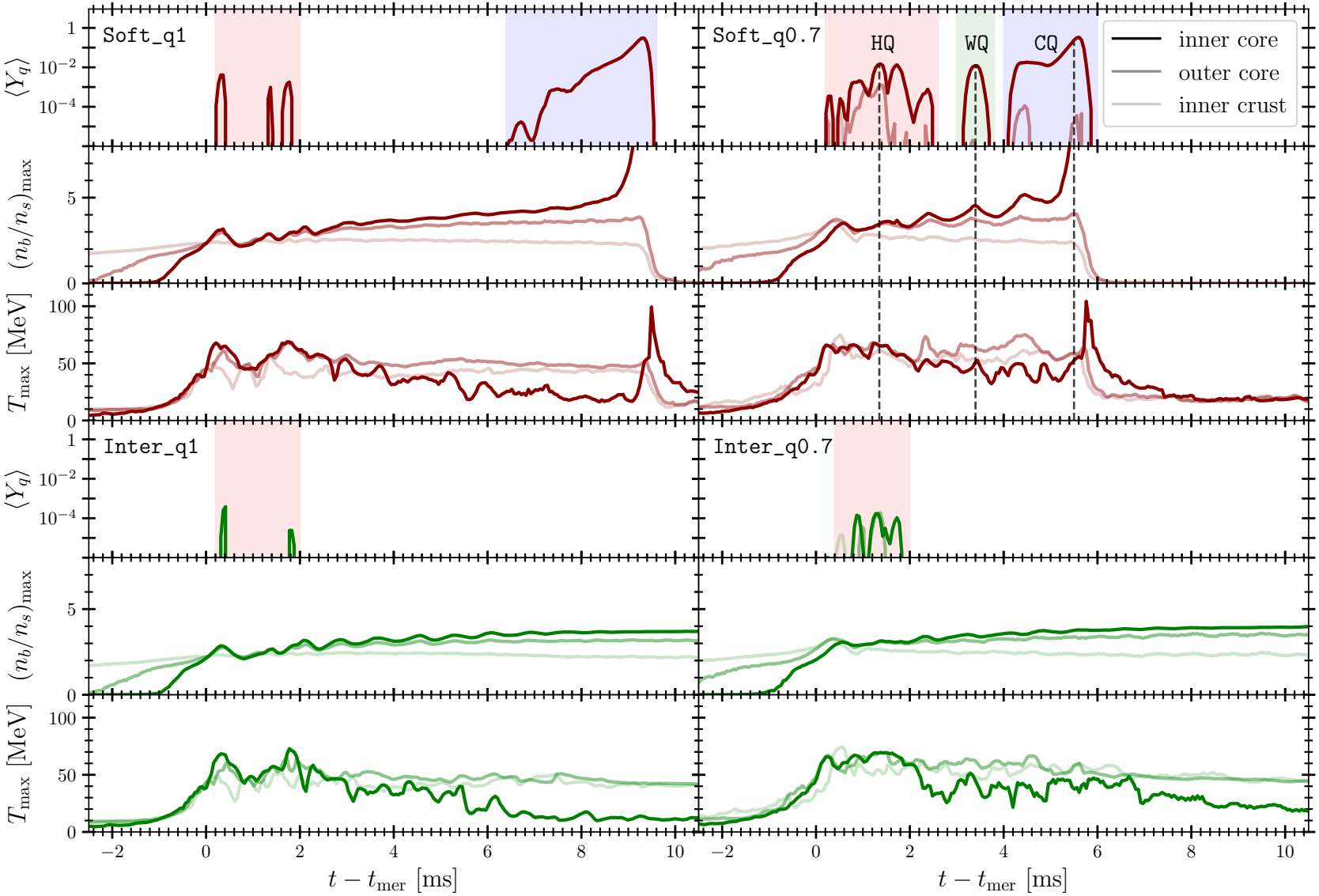


Mechanical Toy Model



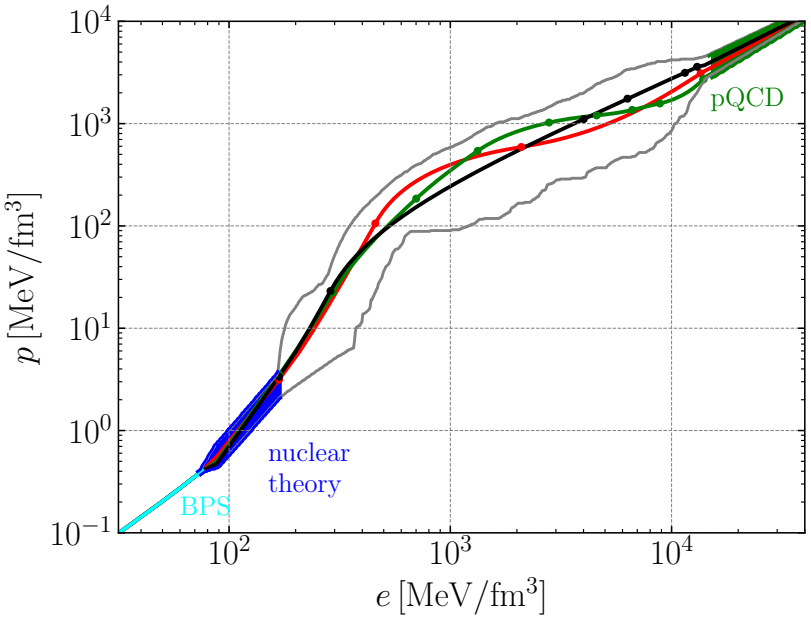
[Takami, Rezzolla, Baiotti arXiv:1412.3240]

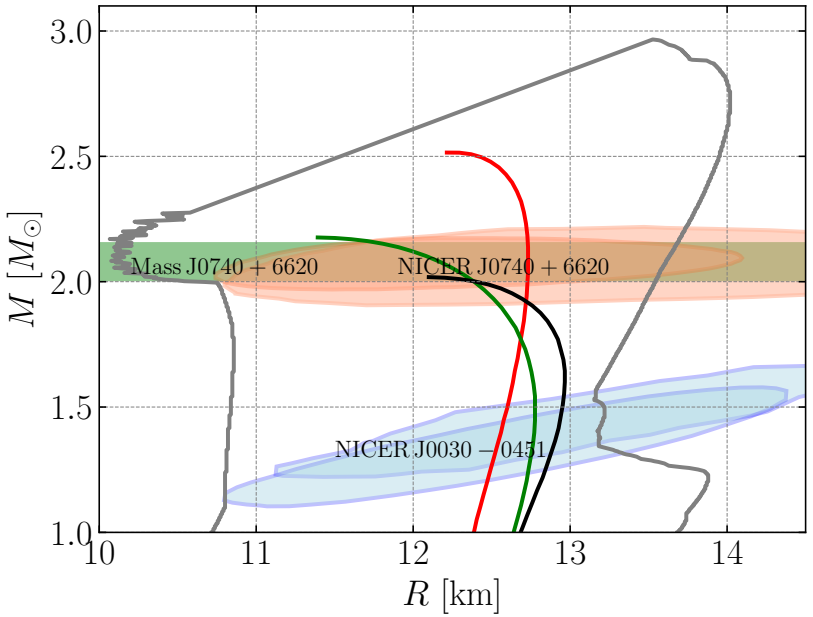

T [MeV]
0
6
12
18
24
30
36
42
Inter, q = 1
t − t_mer = 34.9 ms
n_b = 3 n_s
n_b = 2 n_s
n_b = 1 n_s
y [km]
x [km]
10
5
0
-5
-10
6.4
5.6
4.8
4.0
3.2 n_b/n_s
2.4
1.6
0.8
0.0


Soft_q1
Soft_q0.7
HQ
WQ
CQ
inner core
outer core
inner crust
Inter_q1
Inter_q0.7
$\langle Y_q \rangle$
$(n_b/n_s)_{\max}$
$T_{\max}$ [MeV]
$t - t_{\mathrm{mer}}$ [ms]

Guessing the EOS

- Impose boundary conditions from nuclear theory and pertubative QCD.
- Connect with random, but causal and thermodynamically consistent EoS.
- Constrain with pulsar and binary neutron star observations.





[S. Altiparmak, CE, L. Rezzolla arXiv:2203.14974]

[Lots of older and recent work:

...

Annala, Gorda, Kurkela, Nättilä, Vuorinen arXiv:1903.09121,2105.05132;

Komoltsev, Kurkela arXiv:2111.05350;

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...]

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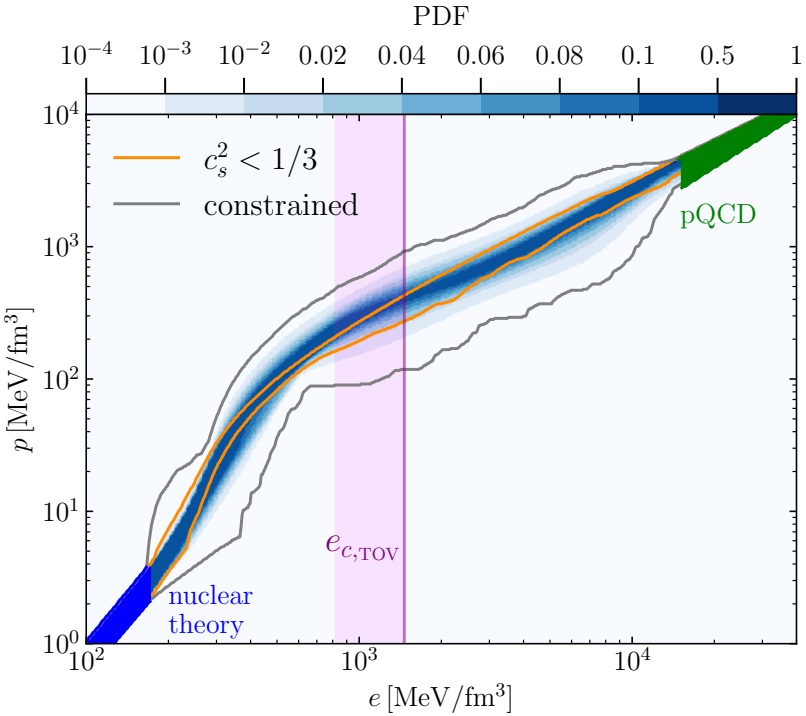

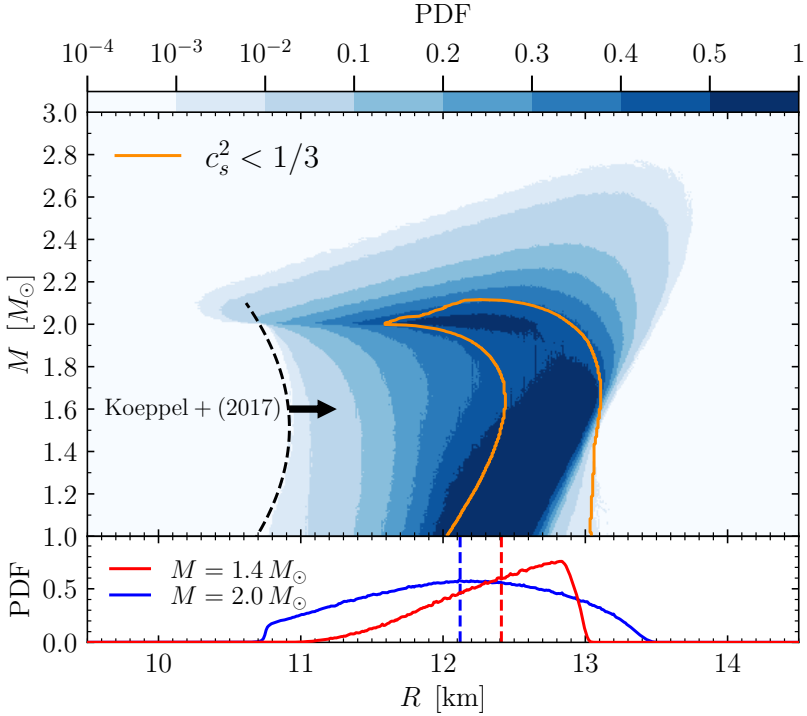

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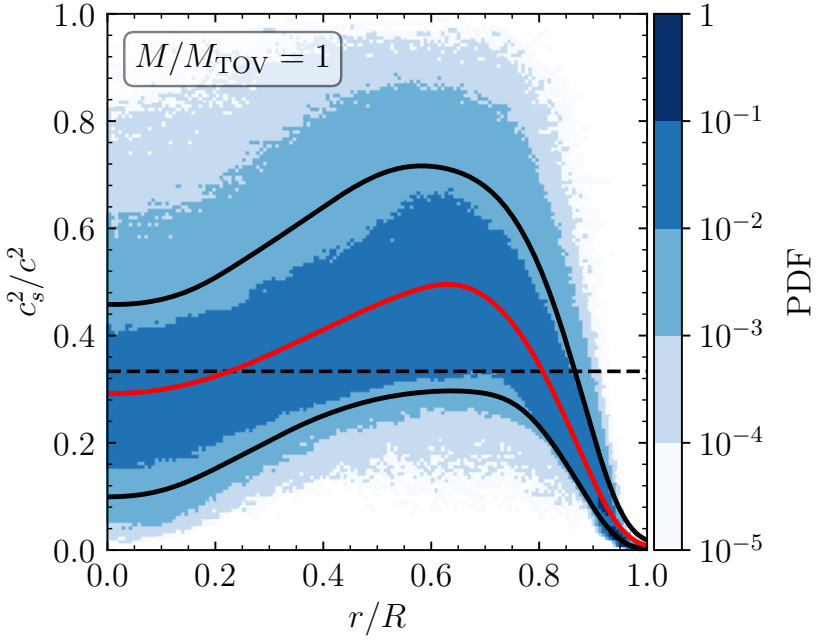
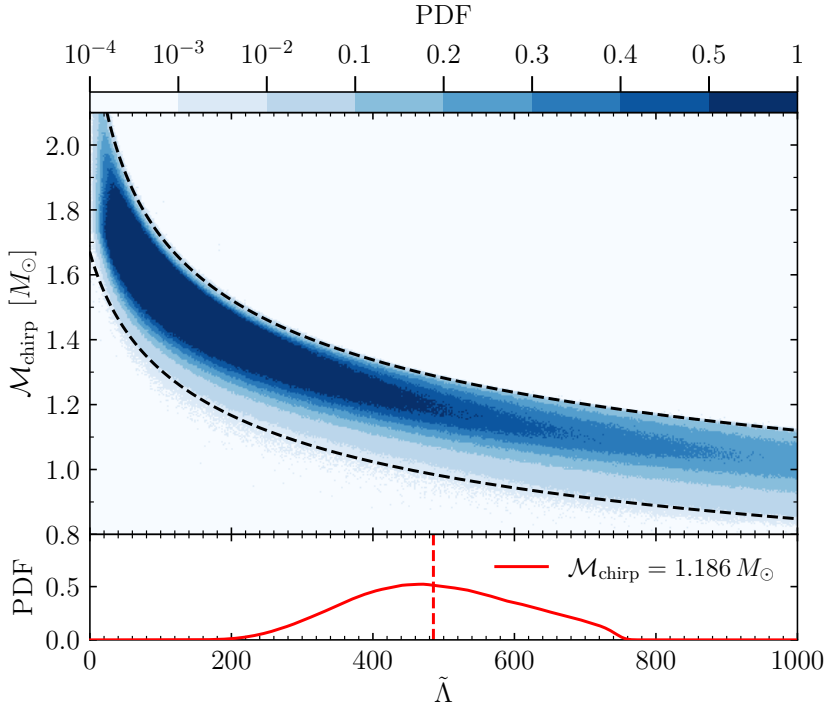
Simple formulas for NS and BNS properties



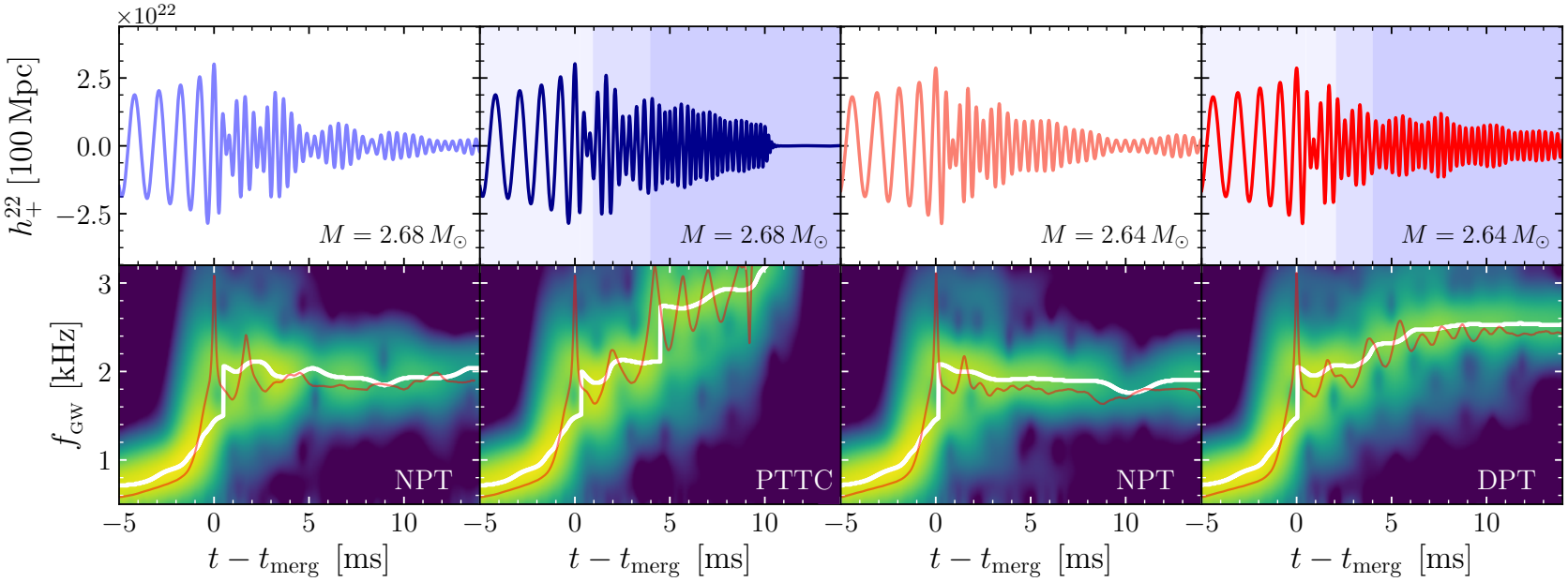


$$c_s^2\left(\frac{r}{R}\right) = \left(\alpha e^{\beta\left(\frac{r}{R}\right)^2} + \gamma e^{\delta\left(\frac{r}{R}-\epsilon\right)^2}\right)\left[1 - \tanh\left(\zeta\frac{r}{R} - \eta\right)\right] \qquad \tilde{\Lambda}_{min(max)} = a + b\,\mathcal{M}_{chirp}^{c}$$

[CE, L. Rezzolla arXiv:2207.04417; S. Altiparmak, CE, L. Rezzolla arXiv:2203.14974]

Phase Transition Signatures in Wavefrom Spectra

- **NPT:** No Phase Transition
- **PTTC:** Phase Transition Triggered Collapse
- **DPT:** Delayed Phase Transition



[Weih, Hanauske, Rezzolla arXiv:1912.09340]